



Exponents and logarithms

IB Mathematics: Applications and Interpretation SL | SL 1.5

50 questions ■ Fluency, reasoning and original exam-style practice

Learning goal	Use integer exponent laws; interpret, evaluate and undo logarithms.
Syllabus link	AI SL 1.5: integer exponents; logarithms in bases 10 and e ; numerical evaluation using technology.
Structure	15 short questions; 15 extended questions; 20 exam-style questions. Worked solutions begin on page 10.
Technology	 GDC marks questions requiring numerical calculator work. Attempt unmarked questions by hand; calculators are permitted throughout Section 3, as in AI SL examinations.

Instructions Show your working on separate paper. Use positive exponents when asked and give exact answers unless a decimal is requested. For numerical GDC answers, use **3 significant figures** unless stated otherwise; keep unrounded values in intermediate steps. Here $\log x = \log_{10} x$ and $\ln x = \log_e x$.

TI-Nspire CX: for $\ln$, press **Ctrl** then e^x ; for $\log$, press **Ctrl** then 10^x .

1 Short response

1.

Read each power carefully.

- (a) Write 5^3 as a product of three factors, then evaluate it.
- (b) Write $(-2)^3$ as a product of three factors, then evaluate it.
- (c) State the base and the exponent in 7^{-2} .

2.

Write each expression as a single power.

- (a) $3^2 \times 3^4$
- (b) $\frac{7^5}{7^2}$
- (c) $(2^3)^4$

3.

Evaluate or simplify. In part (c), $x \neq 0$.

- (a) 12^0
- (b) $(-6)^0$
- (c) $4x^0$

4.

Evaluate, giving exact answers.

- (a) 4^{-2}
- (b) $(-3)^{-3}$
- (c) $\left(\frac{2}{5}\right)^{-2}$

**5.**

Complete the table.

n	3	2	1	0	-1	-2
10^n	1000			1		

6.Write each expression using positive exponents. Take $x \neq 0$.

(a) $(3x)^3$

(b) $2x^{-4}$

(c) $(2x^{-2})^3$

7.

Simplify, using positive exponents. All variables are non-zero.

(a) $\frac{8^2}{8^5}$

(b) $y^{-3}y^7$

(c) $\frac{z^{-2}}{z^{-6}}$

8.

Evaluate the two expressions, then explain why their values differ.

(a) -2^4

(b) $(-2)^4$

9.

Write each statement in logarithmic form.

(a) $10^4 = 10000$

(b) $e^{-2} = \frac{1}{e^2}$

10.

Write each statement in exponential form.

(a) $\log 0.001 = -3$

(b) $\ln u = 5$

11.

Evaluate exactly, without using a calculator.

(a) $\log 100$

(b) $\log 0.0001$

(c) $\ln 1$

**12.**

State whether each value is positive, negative or zero. Justify your answers without calculating the values.

- (a) $\log 0.4$
- (b) $\ln 6$
- (c) $\log 1$

13.

State which expressions have a real value. Give the condition you used.

- (a) $\log(-3)$
- (b) $\ln 0$
- (c) $\ln 0.05$
- (d) $\log 1$

14.  GDC

Evaluate to 3 significant figures.

- (a) $\log 35$
- (b) $\ln 12$
- (c) $\ln 0.08$

15.  GDC

Rewrite each equation in exponential form, then find the unknown to 3 significant figures.

- (a) $\log y = 1.7$
- (b) $\ln z = -0.6$

2 Extended response

16.

Let $E = \frac{(2x^3y^{-1})^2}{4x^{-1}y^{-3}}$, where $x, y \neq 0$.

- (a) Expand the power in the numerator.
- (b) Hence simplify E , using positive exponents.
- (c) Find E when $x = 2$ and $y = 3$.

17.

Let $F = \frac{(3a^{-2}b)^2}{9a^{-1}b^3}$, where $a, b \neq 0$.

- (a) Simplify F , using positive exponents.
- (b) Calculate F when $a = 2$ and $b = 4$.

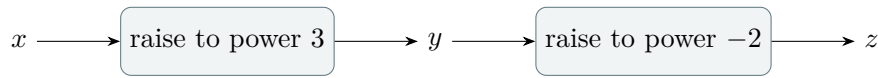
18.

A student writes $x^4 + x^3 = x^7$.

- (a) Use $x = 2$ to show that this statement is false.
- (b) Replace the addition sign with an operation that makes the statement true for every real x .
- (c) State the exponent law that justifies your correction.

**19.**

A non-zero input passes through the following machine.



- (a) Express the final output z as a power of x .
- (b) Find z when $x = 2$.
- (c) Find z when $x = -2$, and explain why the two outputs are equal.

20.

For $t \neq 0$, consider $A = (2t)^0$, $B = 2t^0$ and $C = 2t^{-1}$.

- (a) Simplify A , B and C .
- (b) Find their values when $t = 4$.
- (c) Explain why A and B are different even though both contain a zero exponent.

21.

For each statement, write its equivalent exponential or logarithmic form and find the unknown exactly.

- (a) $10^k = 0.00001$
- (b) $e^t = e^3$
- (c) $\log u = -2$
- (d) $\ln v = -4$

22.  GDC

Consider the number 75.

- (a) Find the integer n for which $10^n < 75 < 10^{n+1}$.
- (b) Hence state two consecutive integers between which $\log 75$ lies.
- (c) Evaluate $\log 75$ to 3 significant figures and check it against your bounds.

23.  GDC

Let $A = \ln 0.04$ and $B = \log 0.04$.

- (a) Predict the sign of each value before using your calculator.
- (b) Evaluate both values to 3 significant figures.
- (c) Explain why the two logarithms need not have equal values.

24.  GDC

Consider $p = -\log(0.0032)$ and $q = \log(-0.0032)$.

- (a) State which expression has a real value, with a reason.
- (b) Evaluate that expression to 3 significant figures.
- (c) Write the equation defining p in exponential form.

**25.**

Complete the table exactly, then explain the relationship between the first and third columns.

x	e^x	$\ln(e^x)$
-2		
0		
3		

26. GDC

Let $u = \ln 18$.

- (a) Find u to 5 decimal places.
- (b) Use the unrounded stored value of u to calculate e^u . Explain the result.
- (c) Instead use $u = 2.89$ and calculate $e^{2.89}$ to 5 decimal places. Explain why this does not give exactly 18.

27.

Evaluate each expression exactly, using inverse operations. Explain each answer.

- (a) $10^{\log 470}$
- (b) $e^{\ln 0.12}$
- (c) $\log(10^{0.65})$

28. GDC

A simplified sound model is $L = 10 \log R$, where L is the sound level in decibels (dB) and $R > 0$ is a dimensionless intensity ratio.

- (a) Calculate L when $R = 250$.
- (b) Find R when $L = 18$.
- (c) Explain why this formula cannot be used with $R = 0$.

29. GDC

The model $\text{pH} = -\log H$ uses a hydrogen-ion concentration $H > 0$, measured in mol dm^{-3} .

- (a) Find the pH when $H = 2.5 \times 10^{-5}$.
- (b) Find H when the pH is 3.2.
- (c) Which solution has the greater concentration? Compare the calculated values.

30.

Find each unknown integer by comparing exponents.

- (a) $2^n \times 2^{-3} = 2^5$
- (b) $(10^m)^{-2} = 10^6$
- (c) $\frac{x^{-4}}{x^k} = x^2$ for every $x > 0$.



3 IB exam-style practice

Original questions in IB style. Calculators are permitted; show method and reasoning.

31. [Maximum mark: 6]

Let $C = 4^3 \times 4^{-5}$ and $D = \frac{(4^{-1})^3}{4^{-5}}$.

- (a) Write C as a single power of 4. [1 mark]
- (b) Find the exact value of C . [1 mark]
- (c) Simplify D and find its value. [2 marks]
- (d) Hence find the exact value of D/C . [2 marks]

32. [Maximum mark: 6]

Let $E = \frac{(2a^2b^{-1})^3}{4a^{-1}b^{-2}}$, where $a, b \neq 0$.

- (a) Simplify E , using positive exponents. [3 marks]
- (b) Find E when $a = 2$ and $b = 8$. [2 marks]
- (c) Explain why the original expression is not defined when $b = 0$. [1 mark]

33. [Maximum mark: 5]

The numbers p and q satisfy $10^p = 0.00001$ and $\ln q = 2$.

- (a) Write down p . [1 mark]
- (b) Write q as an exact exponential expression. [1 mark]
- (c) Write $10^p = 0.00001$ in logarithmic form. [1 mark]
- (d) State which of p and q is negative, explaining your answer. [2 marks]

34.  GDC [Maximum mark: 6]

Let $k = \log 5200$.

- (a) Explain why $3 < k < 4$. [2 marks]
- (b) Calculate k to 3 significant figures. [2 marks]
- (c) Write the defining equation in exponential form. Use your unrounded value of k to check it numerically. [2 marks]

35. [Maximum mark: 6]

The table shows powers of 3.

r	-2	-1	0	1	2
3^r		$\frac{1}{3}$			9

- (a) Complete the three missing entries. [3 marks]
- (b) Find the exact value of $\frac{3^{-2} \times 3^4}{3}$. [2 marks]
- (c) A student claims that $3^{-2} = -9$. Explain the error. [1 mark]

**36.** GDC**[Maximum mark: 6]**

The positive numbers A and B satisfy $\log A = 2.35$ and $\ln B = 2.35$.

- (a) Write both equations in exponential form. [2 marks]
- (b) Calculate A and B . [2 marks]
- (c) Find the ratio A/B , using unrounded values. [2 marks]

37. GDC**[Maximum mark: 6]**

The pH of a solution is modelled by $\text{pH} = -\log H$, where $H > 0$ is its hydrogen-ion concentration in mol dm^{-3} .

- (a) Solution A has $H = 8.0 \times 10^{-6}$. Find its pH. [2 marks]
- (b) Solution B has pH 4.7. Find its concentration. [2 marks]
- (c) Identify the solution with the higher concentration, justifying your answer. [2 marks]

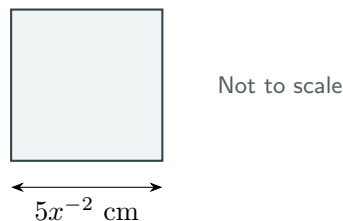
38.**[Maximum mark: 5]**

Consider the three positive numbers 0.02, 1 and 30.

- (a) State the sign of $\log x$ for each number x . [3 marks]
- (b) Would the signs change if $\ln x$ were used? Explain. [1 mark]
- (c) State whether $\ln 0$ has a real value. [1 mark]

39.**[Maximum mark: 6]**

A square has side length $s = 5x^{-2}$ cm, where $x > 0$.



- (a) Express its area A in terms of x , using positive exponents. [2 marks]
- (b) Find its exact area when $x = 2$. [2 marks]
- (c) If x is doubled, find the factor by which the area changes. [2 marks]

40. GDC**[Maximum mark: 6]**

A sound level is modelled by $L = 10 \log R$, where L is measured in dB and $R > 0$ is an intensity ratio.

- (a) Sound A has $R_A = 1200$. Calculate its level. [2 marks]
- (b) Sound B has level 36 dB. Find R_B . [2 marks]
- (c) Find R_B/R_A and interpret this ratio. [2 marks]

**41.****[Maximum mark: 6]**

The first four entries in a pattern are x^{-3} , x^{-2} , x^{-1} and x^0 , where $x > 0$.

- (a) Write these four entries when $x = 2$. [2 marks]
- (b) State the multiplier that takes each entry to the next. [1 mark]
- (c) Write the fifth entry in terms of x . [1 mark]
- (d) Express the product of the first four entries as a single power of x . [2 marks]

42. GDC**[Maximum mark: 6]**

Let $h = \ln u$ and suppose that $\log v = -2.1$.

- (a) Calculate h when $u = 0.06$. [2 marks]
- (b) Calculate v . [2 marks]
- (c) A student says that a negative value of h means that u is negative. Explain why this is false. [2 marks]

43.**[Maximum mark: 5]**

A student writes $\log(-0.1) = -1$ and $\ln 0 = 0$.

- (a) Explain the error in the first statement and correct its argument so that the value is -1 . [2 marks]
- (b) Solve $\ln w = 0$. [1 mark]
- (c) Use exponential form to explain the error in the second statement. [2 marks]

44. GDC**[Maximum mark: 7]**

A measuring device reports an index $S = 2 + \log R$, where its dimensionless input satisfies $R > 0$.

- (a) Calculate S when $R = 320$. [2 marks]
- (b) Find R when $S = 1.4$. [3 marks]
- (c) Explain how an index less than 2 can occur even though the input is positive. [2 marks]

45.**[Maximum mark: 6]**

For $a \neq 0$, consider the following expressions.

- (a) Simplify $(5a^{-2})^0$. [1 mark]
- (b) Simplify $5(a^{-2})^0$. [1 mark]
- (c) Simplify $(5a^{-2})^{-1}$, using positive exponents. [2 marks]
- (d) Find the exact value of the expression in part (c) when $a = -3$. [2 marks]

46. GDC**[Maximum mark: 6]**

A sensor assigns the score $s = \ln C$ to a positive, dimensionless input C .

Input C	10	50	200
Score s			

- (a) Complete the table, giving scores to 3 significant figures. [3 marks]
- (b) Find the input when $s = 4.2$. [2 marks]
- (c) Which of the three listed inputs produces the score closest to 4.2? [1 mark]



47.

[Maximum mark: 7]

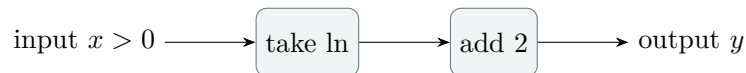
For an integer n , let $M = \frac{3^{2n} \times 3^{-n}}{3^4}$.

- (a) Express M as a single power of 3. [2 marks]
- (b) Find n when $M = 1$. [2 marks]
- (c) Find n when $M = 1/9$. [2 marks]
- (d) Write down M when $n = 0$. [1 mark]

48. GDC

[Maximum mark: 6]

The machine takes a positive input and produces an output.



- (a) Find the output when the input is 0.2. [2 marks]
- (b) Find the input when the output is 5. [3 marks]
- (c) Explain why zero cannot be used as an input. [1 mark]

49.

[Maximum mark: 6]

The positive numbers A and B satisfy $\ln A = -5$ and $\ln B = 2$. Give exact answers throughout.

- (a) Evaluate $\ln(e^{-5})$. [1 mark]
- (b) Evaluate $e^{\ln 7}$. [1 mark]
- (c) Write A and B in exponential form, then express AB as a single power of e . [2 marks]
- (d) Express B/A as a single power of e . [2 marks]

50. GDC

[Maximum mark: 8]

Let $p = \log 64$ and $q = \ln 64$.

- (a) Calculate p and q to 3 significant figures. [2 marks]
- (b) Write down the exact values of 10^p and e^q . [2 marks]
- (c) Find the exact value of $\frac{(2^3)^2}{2^{-1}}$, showing your exponent calculation. [2 marks]
- (d) Explain why $\log(64^{-1})$ is negative, then evaluate it to 3 significant figures. [2 marks]

**Answer key and solution outlines**

Keep exact values until a final approximation is required. Equivalent correct forms are accepted.

Section 1: Short response**1.**

(a) $5^3 = 5 \times 5 \times 5 = 125$.

(b) $(-2)^3 = (-2)(-2)(-2) = -8$.

(c) Base 7; exponent -2 .

2.

(a) $3^{2+4} = 3^6$.

(b) $7^{5-2} = 7^3$.

(c) $2^{3 \times 4} = 2^{12}$.

3.

(a) $12^0 = 1$.

(b) $(-6)^0 = 1$.

(c) $4x^0 = 4 \times 1 = 4$; only x is raised to the power zero.

4.

(a) $4^{-2} = 1/4^2 = 1/16$.

(b) $(-3)^{-3} = 1/(-3)^3 = -1/27$.

(c) $(2/5)^{-2} = (5/2)^2 = 25/4$.

5. In order: $10^3 = 1000$, $10^2 = 100$, $10^1 = 10$, $10^0 = 1$, $10^{-1} = 0.1$, $10^{-2} = 0.01$.

6.

(a) $(3x)^3 = 3^3 x^3 = 27x^3$.

(b) $2x^{-4} = 2/x^4$.

(c) $(2x^{-2})^3 = 2^3 x^{-6} = 8/x^6$.

7.

(a) $8^{2-5} = 8^{-3} = 1/512$.

(b) $y^{-3+7} = y^4$.

(c) $z^{-2-(-6)} = z^4$.

8.

(a) $-2^4 = -(2^4) = -16$.

(b) $(-2)^4 = 16$. The brackets make -2 the base; without brackets, the power is evaluated before the minus sign.

9.

(a) $\log 10000 = 4$.

(b) $\ln(1/e^2) = -2$.

10.

(a) $10^{-3} = 0.001$.

(b) $e^5 = u$.

11.

(a) $\log 100 = 2$, since $10^2 = 100$.

(b) $\log 0.0001 = -4$, since $10^{-4} = 0.0001$.

(c) $\ln 1 = 0$, since $e^0 = 1$.

12.

(a) Negative: $0 < 0.4 < 1$ and the base $10 > 1$.

(b) Positive: $6 > 1$ and the base $e > 1$.

(c) Zero: $10^0 = 1$.

**Section 1: Short response (continued)****13.**

- (a) No real value: the argument -3 is negative.
- (b) No real value: the argument is zero.
- (c) A real value: $0.05 > 0$.
- (d) A real value: $1 > 0$. A real logarithm requires a strictly positive argument.

14.

- (a) $\log 35 = 1.544068 \dots \approx 1.54$.
- (b) $\ln 12 = 2.484906 \dots \approx 2.48$.
- (c) $\ln 0.08 = -2.525728 \dots \approx -2.53$.

15.

- (a) $y = 10^{1.7} = 50.1187 \dots \approx 50.1$.
- (b) $z = e^{-0.6} = 0.548811 \dots \approx 0.549$.

Section 2: Extended response**16.**

- (a) $(2x^3y^{-1})^2 = 4x^6y^{-2}$.
- (b) $E = (4/4)x^{6-(-1)}y^{-2-(-3)} = x^7y$.
- (c) $E = 2^7 \times 3 = 384$.

17.

- (a) $F = \frac{9a^{-4}b^2}{9a^{-1}b^3} = a^{-3}b^{-1} = \frac{1}{a^3b}$.
- (b) $F = 1/(2^3 \times 4) = 1/32$.

18.

- (a) At $x = 2$, the left side is $16 + 8 = 24$, but the right side is 128.
- (b) $x^4 \times x^3 = x^7$.
- (c) When multiplying powers with the same base, add the exponents: $x^m x^n = x^{m+n}$.

19.

- (a) $y = x^3$, so $z = y^{-2} = (x^3)^{-2} = x^{-6} = 1/x^6$.
- (b) $z = 2^{-6} = 1/64$.
- (c) $z = (-2)^{-6} = 1/64$. The even power in the denominator gives the same value for 2 and -2 .

20.

- (a) $A = 1$, $B = 2$, $C = 2/t$.
- (b) $A = 1$, $B = 2$, $C = 1/2$.
- (c) In A , the whole product $2t$ is raised to power zero. In B , only t is raised to power zero; the factor 2 remains.

21.

- (a) $\log 0.00001 = k$, so $k = -5$.
- (b) $\ln(e^3) = t$, so $t = 3$.
- (c) $10^{-2} = u$, so $u = 0.01$.
- (d) $e^{-4} = v$, so $v = e^{-4} = 1/e^4$.

22.

- (a) $10^1 = 10 < 75 < 100 = 10^2$, so $n = 1$.
- (b) $1 < \log 75 < 2$.
- (c) $\log 75 = 1.875061 \dots \approx 1.88$, which lies between 1 and 2.

23.

- (a) Both are negative: the argument is between 0 and 1, and both bases exceed 1.
- (b) $A = -3.218875 \dots \approx -3.22$; $B = -1.397940 \dots \approx -1.40$.
- (c) The bases differ: $\ln$ uses base e , whereas $\log$ uses base 10.

**Section 2: Extended response (continued)****24.**

- (a) Only p has a real value. Its logarithm has positive argument 0.0032; q has a negative argument.
(b) $p = -(-2.494850 \dots) = 2.494850 \dots \approx 2.49$.
(c) $\log 0.0032 = -p$, so $10^{-p} = 0.0032$.

25. Rows: $(-2, e^{-2}, -2)$; $(0, 1, 0)$; $(3, e^3, 3)$. Taking $\ln$ undoes raising e to that power, so $\ln(e^x) = x$.

26.

- (a) $u = 2.890371 \dots \approx 2.89037$.
(b) $e^u = e^{\ln 18} = 18$ (up to calculator precision), because the operations are inverses.
(c) $e^{2.89} \approx 17.99331$ to 5 decimal places. Rounding the exponent changes the input to the exponential calculation.

27.

- (a) 470: $\log 470$ is the exponent that makes a power of 10 equal to 470.
(b) 0.12: raising e to $\ln 0.12$ returns its positive argument.
(c) 0.65: the logarithm asks for the exponent on 10.

28.

- (a) $L = 10 \log 250 = 23.9794 \dots \approx 24.0$ dB.
(b) $18 = 10 \log R \Rightarrow \log R = 1.8 \Rightarrow R = 10^{1.8} = 63.0957 \dots \approx 63.1$.
(c) $\log 0$ has no real value; the argument must be strictly positive.

29.

- (a) $\text{pH} = -\log(2.5 \times 10^{-5}) = 4.602059 \dots \approx 4.60$.
(b) $\log H = -3.2 \Rightarrow H = 10^{-3.2} = 6.30957 \dots \times 10^{-4} \approx 6.31 \times 10^{-4} \text{ mol dm}^{-3}$.
(c) The solution with pH 3.2: $6.31 \times 10^{-4} > 2.5 \times 10^{-5}$.

30.

- (a) $n - 3 = 5$, so $n = 8$.
(b) $-2m = 6$, so $m = -3$.
(c) $-4 - k = 2$, so $k = -6$.

Section 3: IB exam-style practice**31.**

- (a) $C = 4^{3-5} = 4^{-2}$.
(b) $C = 1/16$.
(c) $D = 4^{-3-(-5)} = 4^2 = 16$.
(d) $D/C = 16 \div (1/16) = 256$.

32.

- (a) $E = \frac{8a^6b^{-3}}{4a^{-1}b^{-2}} = 2a^{6-(-1)}b^{-3-(-2)} = 2a^7/b$.
(b) $E = 2 \times 2^7/8 = 32$.
(c) The terms b^{-1} and b^{-2} require division by a power of b , so $b = 0$ is not allowed.

33.

- (a) $p = -5$.
(b) $q = e^2$.
(c) $\log 0.00001 = p$ (or $\log 0.00001 = -5$).
(d) Only p is negative. $p = -5$, whereas $q = e^2 > 0$.

34.

- (a) $10^3 = 1000 < 5200 < 10000 = 10^4$, so the required exponent lies between 3 and 4.
(b) $k = 3.716003 \dots \approx 3.72$.
(c) $10^k = 5200$; entering $10^{\log 5200}$ gives 5200, up to calculator precision.



Section 3: IB exam-style practice (continued)

35.

- (a) The missing entries are $1/9$, 1 and 3 , in that order.
(b) $3^{-2+4-1} = 3^1 = 3$.
(c) A negative exponent means a reciprocal, so $3^{-2} = 1/3^2 = 1/9$; it does not make the value negative.

36.

- (a) $A = 10^{2.35}$ and $B = e^{2.35}$.
(b) $A = 223.872 \dots \approx 224$; $B = 10.4855 \dots \approx 10.5$.
(c) $A/B = 21.350495 \dots \approx 21.4$.

37.

- (a) $\text{pH}_A = -\log(8.0 \times 10^{-6}) = 5.096910 \dots \approx 5.10$.
(b) $\log H_B = -4.7 \Rightarrow H_B = 10^{-4.7} = 1.99526 \dots \times 10^{-5} \approx 2.00 \times 10^{-5} \text{ mol dm}^{-3}$.
(c) Solution B, since $1.99526 \dots \times 10^{-5} > 8.0 \times 10^{-6}$.

38.

- (a) In order: negative, zero, positive.
(b) No. Both bases, 10 and e , are greater than 1 , so the same sign rules apply.
(c) No; a real logarithm requires a strictly positive argument.

39.

- (a) $A = s^2 = (5x^{-2})^2 = 25x^{-4} = 25/x^4 \text{ cm}^2$.
(b) $A = 25/2^4 = 25/16 \text{ cm}^2$.
(c) $A_{\text{new}} = 25/(2x)^4 = 25/(16x^4) = A/16$: the area is multiplied by $1/16$.

40.

- (a) $L_A = 10 \log 1200 = 30.7918 \dots \approx 30.8 \text{ dB}$.
(b) $\log R_B = 3.6 \Rightarrow R_B = 10^{3.6} = 3981.07 \dots \approx 3.98 \times 10^3$.
(c) $R_B/R_A = 3981.07 \dots / 1200 = 3.31755 \dots \approx 3.32$. Sound B has about 3.32 times the intensity of sound A.

41.

- (a) $1/8$, $1/4$, $1/2$, 1 .
(b) Multiply by x .
(c) $x^1 = x$.
(d) $x^{-3-2-1+0} = x^{-6}$.

42.

- (a) $h = \ln 0.06 = -2.813410 \dots \approx -2.81$.
(b) $v = 10^{-2.1} = 0.00794328 \dots \approx 0.00794$.
(c) For a negative natural logarithm, $0 < u < 1$. Here $u = 0.06 > 0$, but $h < 0$. The argument must still be positive.

43.

- (a) A negative argument has no real logarithm. The correct statement is $\log 0.1 = -1$, since $10^{-1} = 0.1$.
(b) $w = e^0 = 1$.
(c) $\ln 0 = 0$ would mean $e^0 = 0$, but $e^0 = 1$. Thus $\ln 1 = 0$; $\ln 0$ is undefined over the reals.

44.

- (a) $S = 2 + \log 320 = 4.505149 \dots \approx 4.51$.
(b) $1.4 = 2 + \log R \Rightarrow \log R = -0.6 \Rightarrow R = 10^{-0.6} = 0.251188 \dots \approx 0.251$.
(c) When $0 < R < 1$, $\log R < 0$, so $2 + \log R < 2$. A positive argument can give a negative logarithm.

45.

- (a) $(5a^{-2})^0 = 1$, since its base is non-zero.
(b) $5(a^{-2})^0 = 5 \times 1 = 5$.
(c) $(5a^{-2})^{-1} = 1/(5a^{-2}) = a^2/5$.
(d) $(-3)^2/5 = 9/5$.

46.

- (a) The scores are 2.30 , 3.91 and 5.30 , from $\ln 10$, $\ln 50$ and $\ln 200$.
(b) $C = e^{4.2} = 66.6863 \dots \approx 66.7$.
(c) $C = 50$: its score $3.91202 \dots$ is closest to 4.2 .

**Section 3: IB exam-style practice (continued)****47.**

- (a) $M = 3^{2n-n-4} = 3^{n-4}$.
- (b) $3^0 = 1$, so $n - 4 = 0$ and $n = 4$.
- (c) $1/9 = 3^{-2}$, so $n - 4 = -2$ and $n = 2$.
- (d) $M = 3^{-4} = 1/81$.

48.

- (a) $y = \ln 0.2 + 2 = -1.609437 \dots + 2 = 0.390562 \dots \approx 0.391$.
- (b) $\ln x + 2 = 5 \Rightarrow \ln x = 3 \Rightarrow x = e^3 = 20.0855 \dots \approx 20.1$.
- (c) The first operation would require $\ln 0$, which has no real value.

49.

- (a) -5 , since $\ln$ and the exponential with base e are inverse operations.
- (b) 7 .
- (c) $A = e^{-5}$ and $B = e^2$, so $AB = e^{-5+2} = e^{-3}$.
- (d) $B/A = e^{2-(-5)} = e^7$.

50.

- (a) $p = 1.806179 \dots \approx 1.81$; $q = 4.158883 \dots \approx 4.16$.
- (b) $10^p = 64$ and $e^q = 64$.
- (c) $2^{3 \times 2 - (-1)} = 2^7 = 128$.
- (d) $64^{-1} = 1/64$ lies between 0 and 1, so its base-10 logarithm is negative. $\log(1/64) = -1.806179 \dots \approx -1.81$.