



TI-Nspire CX skills workbook

IB Mathematics: Analysis and Approaches HL

Topics 2 & 5 | 40 questions | Every listed skill practised at least twice

Learning goal	Choose, carry out and check the calculator operations used in Functions and Calculus.
Syllabus link	AA Topic 2: SL 2.1–2.11 and AHL 2.12–2.16; Topic 5: SL 5.1–5.11 and AHL 5.12–5.19.
Structure	20 paired sections, 40 questions, a two-encounter checklist and a matching answer key in the full edition.
Technology	TI-Nspire CX non-CAS ; Scratchpad for calculations and graphs. A document is used only for spreadsheets (Q33–34). Use Radian mode.

How to use this workbook

Work on separate paper. Read the method reminder, complete the first question, then try its partner independently. For stronger retrieval practice, return to the partner in a later lesson. This is a multi-session workbook, not a single lesson test.

Unless stated otherwise, give numerical answers to **three significant figures**; retain unrounded values for later calculations. Keep exact values when they are known. Include sketches, equations, integral setups and units where requested. Use **Scratchpad Calculate** and **Scratchpad Graph** throughout. Before a new section, record the results you need, then clear Scratchpad to remove old definitions. **Only the Lists & Spreadsheet work in Questions 33–34 requires a document.** Function-value tables opened from a graph remain in Scratchpad.

Quick contents

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Scope. The syllabus specifies mathematical uses of technology, not an exhaustive list of device buttons. The checklist maps those uses to a comprehensive CX workflow. Sliders and slope fields are useful learning tools; they are not additional AA syllabus requirements. Algebraic proofs, exact differentiation, integration techniques and series derivations still need written mathematics. Menu wording can vary with operating system and language; use the named tool or Catalog entry rather than relying on menu numbers.



Skills checklist 1 of 2

Tick each encounter only after carrying out the operation yourself. Every row identifies two distinct questions; later mixed questions provide additional practice.

ID	Calculator skill	First	Second
01	Open, clear and switch between Scratchpad views	☐ Q1	☐ Q2
02	Radian mode; display precision; decimal evaluation	☐ Q1(a-c)	☐ Q2(a-c)
03	Define functions; use expression templates; evaluate inputs	☐ Q1(a-b)	☐ Q2(a-b)
04	Store and recall full-precision values; reuse history	☐ Q1(b-c)	☐ Q2(b-c)
05	Enter and graph piecewise definitions	☐ Q1(e)	☐ Q2(e)
06	Enter several functions and switch between them	☐ Q3(a,d)	☐ Q4(a,d)
07	Set window bounds and scales; zoom and restore	☐ Q3(a-b)	☐ Q4(a-b)
08	Graph Trace at a specified input	☐ Q3(b)	☐ Q4(a)
09	Automatic tables: choose Start and Step	☐ Q3(c)	☐ Q4(c)
10	Ask tables: enter chosen inputs	☐ Q3(d)	☐ Q4(d)
11	Graph sums and differences of stored functions	☐ Q3(d)	☐ Q4(d)
12	Restrict a graph to a given domain	☐ Q5	☐ Q6
13	Find all zeros and axis intercepts	☐ Q5(a)	☐ Q6(a)
14	Locate local maxima and minima with bounds	☐ Q5(b)	☐ Q6(b)
15	Compare endpoints; absolute extrema and range	☐ Q5(c)	☐ Q6(c)
16	Transfer a calculator graph to a labelled sketch	☐ Q5(d)	☐ Q6(d)
17	Find every intersection of two graphs	☐ Q7(a)	☐ Q8(a)
18	nSolve with guesses and lower/upper bounds	☐ Q7(b)	☐ Q8(b)
19	Check roots by substitution; understand one-root output	☐ Q7(d)	☐ Q8(d)
20	Use graph differences to solve inequalities	☐ Q7(c)	☐ Q8(c)
21	Real/complex polynomial roots; coefficient lists	☐ Q9(a)	☐ Q10(a)
22	Solve linear systems for function coefficients	☐ Q9(c)	☐ Q10(c)
23	Investigate vertical and horizontal asymptotes	☐ Q11(a)	☐ Q12(a)
24	Investigate oblique asymptotes via differences	☐ Q11(b)	☐ Q12(b)
25	Detect domain holes and screen artefacts	☐ Q11(c)	☐ Q12(c)
26	Evaluate and graph compositions on valid domains	☐ Q13(a)	☐ Q14(a)
27	Graph inverse pairs; check compositions and restrictions	☐ Q13(b-c)	☐ Q14(b-c)
28	Investigate odd/even symmetry and period	☐ Q13(d-e)	☐ Q14(d-e)
29	Graph combined transformations, including $f(ax+b)$	☐ Q15(a)	☐ Q16(a)
30	Distinguish $\text{abs}(f(x))$ from $f(\text{abs}(x))$	☐ Q15(b)	☐ Q16(b)
31	Graph $1/f(x)$ and $[f(x)]$ squared	☐ Q15(c)	☐ Q16(c)
32	Solve modulus equations and inequalities	☐ Q15(d)	☐ Q16(d)



Skills checklist 2 of 2

Tick each encounter only after carrying out the operation yourself. Every row identifies two distinct questions; later mixed questions provide additional practice.

ID	Calculator skill	First	Second
33	Use sliders to investigate parameters and root counts	☐ Q15(e)	☐ Q16(e)
34	Estimate limits using two-sided tables and graphs	☐ Q17(a,c)	☐ Q18(a,c)
35	Evaluate first-principles difference quotients	☐ Q17(b)	☐ Q18(b)
36	Recognise undefined values and numerical limitations	☐ Q17(c-d)	☐ Q18(c-d)
37	Numerical first and second derivatives at a point	☐ Q19(a)	☐ Q20(a)
38	Graph numerical first and second derivatives	☐ Q19(b)	☐ Q20(b)
39	Use Analyze Graph dy/dx; plot tangent and normal	☐ Q19(c)	☐ Q20(c)
40	Solve an equation involving a derivative	☐ Q19(d)	☐ Q20(d)
41	Stationary points and increasing/decreasing intervals	☐ Q21(a-b)	☐ Q22(a-b)
42	Inflexions and concavity; check sign changes	☐ Q21(b-c,e)	☐ Q22(b-c,e)
43	Optimise on a closed interval, including endpoints	☐ Q21(d)	☐ Q22(d)
44	Evaluate definite integrals in both Scratchpad views	☐ Q23(a-b)	☐ Q24(a-b)
45	Total area: absolute values and split integrals	☐ Q23(c-d)	☐ Q24(c-d)
46	Area between curves, including changing order	☐ Q25(a-b)	☐ Q26(a-b)
47	Integrate with respect to y; right minus left	☐ Q25(c-d)	☐ Q26(c-d)
48	Graph and solve for an unknown integral bound	☐ Q27(a-b)	☐ Q28(a-b)
49	Solve for a parameter multiplying an integrand	☐ Q27(c)	☐ Q28(c)
50	Velocity, speed, acceleration and direction changes	☐ Q29(a-b,e)	☐ Q30(a-b,e)
51	Displacement, distance and position from velocity	☐ Q29(c-d)	☐ Q30(c-d)
52	Volumes about x-axis and y-axis	☐ Q31(a-b)	☐ Q32(a-b)
53	Washer volumes: outer squared minus inner squared	☐ Q31(c-d)	☐ Q32(c-d)
54	Euler spreadsheet recurrence; fill relative references	☐ Q33(a-b)	☐ Q34(a-b)
55	Euler command; distinguish output step and substeps	☐ Q33(b)	☐ Q34(b)
56	Graph an ODE and slope field in Scratchpad	☐ Q33(d)	☐ Q34(d)
57	Compare Euler step sizes and calculate error	☐ Q33(c)	☐ Q34(c)
58	Check implicit differentiation via an explicit branch	☐ Q35(a)	☐ Q36(a)
59	Evaluate related rates using a numerical derivative	☐ Q35(b)	☐ Q36(b)
60	Check antiderivatives and analytical definite integrals	☐ Q35(c-d)	☐ Q36(c-d)
61	Evaluate a higher derivative already derived by hand	☐ Q35(e)	☐ Q36(e)
62	Evaluate Maclaurin polynomials and approximation errors	☐ Q37(a-b)	☐ Q38(a-b)
63	Solve an error-tolerance inequality graphically	☐ Q37(c-d)	☐ Q38(c-d)
64	Use numerical limits to check a series coefficient	☐ Q37(e)	☐ Q38(e)



1. Set up, define and evaluate

Syllabus: 2.2, 2.9; supporting skills

Calculator method

Open **Scratchpad Calculate**; press the **Scratchpad** key to switch between Calculate and Graph. In **Home** → **Settings** → **Document Settings**, choose **Radian** and **Float 6** (or more), then **Make Default** so the settings apply to Scratchpad. This menu name does not mean you need to create a document. Use **f1(x):=** to define a function, and **a:=** to store a value. Use the mathematical e^x , $\ln$ and fraction templates; use **Ctrl+Enter** when you need a decimal. After recording any results you need, use **Doc** → **Clear Scratchpad** for a fresh start; then re-enter the next functions. Calculate and Graph share Scratchpad variables. For different formulas on different intervals, use the **piecewise template** and enter a condition beside each expression. The log template accepts a base; inverse trigonometric functions are in the trig menu (inverse sine is not $1/\sin$).

1. Open **Scratchpad Calculate**. Start with a clear Scratchpad after recording any earlier work. Confirm that $\sin(\pi/6) = 0.5$. Define $f_1(x) = \frac{3x^2 - 2}{x + 1}$.
 - (a) Calculate $f_1(0.4)$ and $f_1(-2)$, using correct brackets. Also evaluate $\log_2 7$ and $\arcsin(0.5)$.
 - (b) Store $a = \sqrt{2}$. Evaluate $f_1(a)$ without rounding a first. Record the result to three significant figures.
 - (c) Find the difference between this result and $f_1(1.41)$, using the unrounded result from your history. Explain why storing a value is useful.
 - (d) Switch to **Scratchpad Graph** and plot f_1 . Record the excluded input. Switch back to Calculate and check that $f_1(0.4)$ is still available.
 - (e) Use the piecewise template to define $h(x) = x + 1$ for $x < 0$ and $h(x) = x^2$ for $x \geq 0$. Graph it and evaluate $h(-1), h(0), h(1)$.
2. Record your previous results, then use **Doc** → **Clear Scratchpad**. Return to **Scratchpad Calculate**. Recheck Radian mode and your display precision. Define $f_1(x) = e^{-x} + \ln(x + 2)$.
 - (a) Calculate $f_1(0.6)$ and $f_1(0)$. Check $\cos(\pi/3) = 0.5$. Also evaluate $\log_3 10$ and $\arccos(0.5)$.
 - (b) Store $b = \sqrt{3}$ and evaluate $f_1(b)$ to three significant figures, retaining full precision internally.
 - (c) Find the difference between this value and $f_1(1.73)$. Use calculation history rather than retyping a rounded answer.
 - (d) Switch to **Scratchpad Graph**, plot the function and state its domain. Return to Calculate and evaluate $f_1(0.6)$ again without redefining it.
 - (e) Define and graph $h(x) = 2 - x$ for $x < 1$ and $h(x) = x^2$ for $x \geq 1$, using the piecewise template. Evaluate $h(0), h(1), h(2)$.

2. Windows, traces and tables

Syllabus: 2.2–2.4

Calculator method

In Scratchpad Graph, use **Window/Zoom** → **Window Settings**; record $x_{\min}, x_{\max}, y_{\min}, y_{\max}$. Use **Trace** → **Graph Trace**, then type an input. **Ctrl+T** opens a function-value table in Scratchpad; focus the table and use **Edit Table Settings** to set Start and Step. Set Independent to **Ask** for chosen inputs. Use **Zoom In/Out** to inspect detail, then restore the recorded window.

3. Plot $f_1(x) = 0.02(x - 20)^2 - 3$ and $f_2(x) = 0.1x - 2$.
 - (a) Set the window to $-10 \leq x \leq 50, -5 \leq y \leq 20$. Explain what a default $[-10, 10]$ horizontal window would hide.
 - (b) Use Graph Trace to find $f_1(22.5)$. Zoom in on the vertex, then restore the original window.
 - (c) Make an automatic table starting at 15 with step 2.5. Copy five rows for f_1 and f_2 .
 - (d) Switch to Ask and find $f_1(18.3)$. Plot $f_3 = f_1 + f_2$ and $f_4 = f_1 - f_2$; calculate both at $x = 20$.



4. Plot $f_1(x) = e^{-0.4x} \cos(2x)$ and $f_2(x) = 0.2$ in Radian mode.
- (a) Choose $0 \leq x \leq 12$, $-1.2 \leq y \leq 1.2$. Use Graph Trace to evaluate $f_1(1.25)$.
 - (b) Zoom in on the first trough, then restore the window. Explain why the vertical scale matters.
 - (c) Set an automatic table with Start 0 and Step 0.5. Copy the first five rows for both functions.
 - (d) Use Ask for $x = 1.3$. Plot $f_1 + f_2$ and $f_1 - f_2$ and evaluate them at $x = 0$.

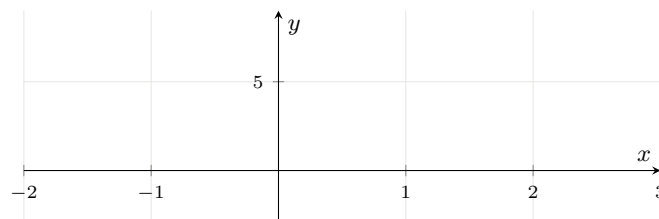
3. Restrictions, intercepts and extrema

Syllabus: 2.2–2.4, 2.6, 2.12; 5.8

Calculator method

For a restriction, enter `piecewise(x^3-3*x,x>=-2 and x<=2.5)`; the function is undefined outside the condition. In Scratchpad Graph, under **Analyze Graph**, use **Zero**, **Minimum** or **Maximum**; choose bounds around one feature. Find the y -intercept by evaluating at 0. For a closed interval, compare all turning-point values with both endpoint values. Transfer key coordinates to a labelled sketch.

5. Plot $f(x) = x^3 - 3x$ restricted to $-2 \leq x \leq 2.5$.
- (a) Use Zero to find every x -intercept; find the y -intercept.
 - (b) Use Maximum and Minimum to locate both local turning points. State the axis-coordinate values, not just the heights.
 - (c) Evaluate both endpoints. Hence state the absolute maximum, absolute minimum and range on the given domain.
 - (d) Make a labelled sketch on the axes below, including the restriction endpoints. Distinguish local and absolute extrema.



6. Plot $g(x) = x^4 - 4x^2 + 1$ restricted to $-2 \leq x \leq 2.2$.
- (a) Find all x -intercepts and the y -intercept.
 - (b) Locate every local minimum and maximum. Widen or refine the window to avoid missing a feature.
 - (c) Evaluate both endpoints. State the absolute extrema and range.
 - (d) Make a labelled sketch on your own axes. Explain why the local maximum is not the absolute maximum on this domain.



4. All intersections, numerical roots and inequalities

Syllabus: 2.4, 2.10, 2.15

Calculator method

Plot both sides before solving. Use **Analyze Graph** → **Intersection**, bracketing one crossing at a time. Cross-check with `nSolve(equation,x=guesst,lower,upper)`. Store each root directly, for example `r:=nSolve(...)`. One call returns one real solution; failure to find a root does not prove there is none. Test one point in each interval for an inequality, and respect domain exclusions.

7. Consider $e^x = 3x^2$ on $-2 \leq x \leq 5$.
- Plot both sides and find every intersection. Record both coordinates and a suitable window.
 - Use `nSolve` three times, with guesses and bounds taken from your graph. Store the roots as $r_1 < r_2 < r_3$.
 - Plot $e^x - 3x^2$ and use its sign to solve $e^x \geq 3x^2$ on the stated interval. Include equality endpoints.
 - Substitute each stored root into $e^x - 3x^2$ to check the residual. Why is a single successful `nSolve` call insufficient here?
8. Consider $\ln(x+3) = x^2$ on $-3 < x \leq 3$.
- State the domain restriction. Plot both sides and find all intersection coordinates.
 - Use `nSolve` with a separate guess and bounding interval for each root; store the results.
 - Solve $\ln(x+3) > x^2$ on the stated domain, using the graph of the difference and interval test values.
 - Check each root by substitution. Explain why entering $x = -3$ or reporting rounded roots as exact boundary values is inappropriate.

5. Polynomial roots and linear systems

Syllabus: 2.1, 2.6–2.7, 2.12; complex-root link to 1.14

Calculator method

Use **Algebra** → **Polynomial Tools** → **Find Roots of Polynomial**, or `polyRoots({coefficients})` in descending powers, including zeros. For coefficient problems, form equations first, then use **Algebra** → **Solve System of Linear Equations** and set the number of equations and variable order. This opens `linSolve`; a list of outputs follows the entered variable order. For complex roots use `cPolyRoots` from the Catalog; choose Rectangular complex format for $a + bi$ output.

9. Use polynomial and simultaneous-equation tools.
- Find all real roots of $x^4 - 5x^2 + 4 = 0$. Write the coefficient list you entered. Verify each root in the original polynomial. Then compare `polyRoots({1,0,1})` with `cPolyRoots({1,0,1})`.
 - Plot the polynomial and confirm its intercepts. Solve $x^4 - 5x^2 + 4 \leq 0$ using the graph.
 - A quadratic $p(x) = ax^2 + bx + c$ passes through (0, 2), (1, 0) and (3, 2). Write three linear equations in a, b, c , then solve them with the system tool.
 - Plot your quadratic and evaluate it at all three given inputs to verify the fit. State its vertex.
10. Record your previous results, then clear Scratchpad before this pair of calculations.
- Find all real roots of $x^3 - 2x^2 - 5x + 6 = 0$. State your coefficient list and verify the results. Then compare `polyRoots({1,0,4})` with `cPolyRoots({1,0,4})`.
 - Plot the cubic and solve $x^3 - 2x^2 - 5x + 6 > 0$.
 - A quadratic $q(x) = ax^2 + bx + c$ passes through (−1, 6), (1, 2) and (2, 3). Set up and solve the three equations for its coefficients.
 - Verify all three points by evaluation and graphing. Use Minimum to find its vertex.



6. Asymptotes, exclusions and misleading screens

Syllabus: 2.4, 2.8, 2.13; 5.12–5.13

Calculator method

Use a table on both sides of a suspected vertical asymptote. Test large positive and negative inputs for horizontal behaviour. For an oblique candidate $y = mx + c$, evaluate $f(x) - (mx + c)$ for large $|x|$. Inspect the original denominator: a cancelled factor can produce a hole, which the screen may conceal. A plotted near-vertical segment is not an additional part of the curve.

11. Investigate three rational functions.

- Plot $f(x) = \frac{2x+1}{x-2}$. Evaluate at 1.99, 2.01, -100 and 100 . Find both intercepts and state its vertical and horizontal asymptotes.
- Plot $g(x) = \frac{x^2+1}{x-1}$ together with $y = x + 1$. Evaluate $g(x) - (x + 1)$ at $x = -100, 100$. State all its asymptotes and real intercepts.
- Plot $h(x) = \frac{x^2-1}{x-1}$. Evaluate at 0.99, 1, 1.01. Explain the missing point and why it is not a vertical asymptote. Sketch h , marking the hole.

12. Repeat the investigation with new functions.

- Plot $f(x) = \frac{3x-2}{x+1}$. Evaluate at -1.01 , -0.99 , -100 , 100 . Find intercepts and vertical and horizontal asymptotes.
- Compare $g(x) = \frac{x^2+4}{x+2}$ with $y = x - 2$. Evaluate their difference at $x = -100, 100$. State all asymptotes and real intercepts.
- Investigate $h(x) = \frac{x^2-4}{x+2}$ near $x = -2$, using $-2.01, -2, -1.99$. Sketch the graph with its missing point. Explain why a screen image alone may be misleading.

7. Composition, inverse, symmetry and period

Syllabus: 2.5, 2.14

Calculator method

Define functions separately, then enter compositions such as $\mathbf{f1(f2(x))}$. For an inverse, first find a valid formula and domain by hand; graph it together with the original and $y = x$. Compare $\mathbf{f1(-x)}$ with $f_1(x)$ and $-f_1(x)$ for symmetry. Use repeated matching features to estimate a period, then check the expression. Numerical agreement supports a conjecture; it is not a proof.

13. Let $f(x) = x^2 - 1$, $x \geq 0$, and $g(x) = 2x + 3$.

- Define both functions. Evaluate $f(g(2))$ and $g(f(2))$. Plot both compositions on their valid domains.
- Find f^{-1} by hand. Graph f , f^{-1} and $y = x$; state the domain and range of each inverse pair.
- Check $f^{-1}(f(2))$ and $f(f^{-1}(3))$. Explain why f^{-1} is not $1/f$.
- Use graphs and values at ± 0.7 to investigate whether $p(x) = x^4 - 2x^2$ is even or odd. Check algebraically.
- Plot $r(x) = \cos(2x)$. Use two consecutive maxima to find its period.



14. Let $f(x) = (x - 1)^2$, $x \geq 1$, and $g(x) = 3x - 2$.
- (a) Evaluate $f(g(2))$ and $g(f(2))$ and plot the compositions on their valid domains.
 - (b) Find and graph f^{-1} , with f and $y = x$. State both domain/range pairs.
 - (c) Check $f^{-1}(f(3))$ and $f(f^{-1}(4))$. Compare $f^{-1}(4)$ with $1/f(4)$.
 - (d) Use graphs and values at ± 0.7 to classify $p(x) = x^3 - x$ as even, odd or neither; justify algebraically.
 - (e) Plot $r(x) = \sin(3x)$ and determine its period from consecutive maxima.

8. Transformations, modulus and parameters

Syllabus: 2.11, 2.16

Calculator method

Use the stored parent function in every expression: `abs(f1(x))`, `f1(abs(x))`, `1/f1(x)`, and `(f1(x))^2`. Compare graphs in a common window, hiding curves when crowded. Create a parameter slider through **Actions** → **Insert Slider** (or accept automatic slider creation). Sliders suggest values; a numerical or algebraic check is still needed.

15. Define the unrestricted parent $f(x) = x^2 - 1$.
- (a) Plot f , $2f(x - 1) + 3$ and $f(2x + 1)$. Track the parent point $(2, 3)$ onto each transformed graph.
 - (b) Plot $|f(x)|$ and $f(|x|)$ separately. Compare their values at $x = -2$ and $x = 0$; explain the difference between the two operations.
 - (c) Plot $1/f(x)$ and $[f(x)]^2$. State their zeros and domain exclusions.
 - (d) Solve $|x^2 - 1| = 0.5$ and $|x^2 - 1| < 0.5$ graphically.
 - (e) Create a slider for k in $y = f(x) + k$. Test $k = -1, 1, 2$; find the value at which two real roots merge into one.
16. Define the unrestricted parent $f(x) = x^3 - 1$.
- (a) Plot f , $-f(x + 2) + 1$ and $f(3x - 2)$. Track $(2, 7)$ onto both transformed graphs.
 - (b) Compare $|f(x)|$ and $f(|x|)$ at $x = -2$ and $x = 0$, using both graphs and evaluation.
 - (c) Plot $1/f(x)$ and $[f(x)]^2$. State zeros and exclusions.
 - (d) Solve $|x^3 - 1| = 2$ and $|x^3 - 1| \leq 2$ graphically.
 - (e) Create a slider for k in $y = |f(x)| - k$. Test $k = -1, 0, 1$ and describe how many real roots occur.

9. Limits and difference quotients

Syllabus: 5.1, 5.12–5.13

Calculator method

Use Ask tables for inputs approaching a point from both sides. Avoid substituting the point into a quotient that is undefined there. To estimate $f'(a)$ from first principles, evaluate $[f(a + h) - f(a)]/h$ for positive and negative h . Very tiny steps can amplify rounding error. A finite table suggests a limit; use analytical reasoning when the question requires proof.

17. Investigate numerical limits in Radian mode.
- (a) Tabulate $\sin x/x$ at $x = \pm 0.1, \pm 0.01, \pm 0.001$. Estimate its limit as $x \rightarrow 0$. Explain why direct substitution fails.
 - (b) For $f(x) = x^3$, define $D(h) = [f(2 + h) - f(2)]/h$. Tabulate the same six nonzero values of h and estimate $f'(2)$.
 - (c) Plot $\sin x/x$ near 0 and note its missing point. Compare this with $|x|/x$ near 0 using a two-sided table. Which limit does not exist?
 - (d) Explain why an apparently smooth screen image is insufficient to establish continuity at 0.



18. Investigate a second collection of limits.

- (a) Tabulate $(e^x - 1)/x$ at $x = \pm 0.1, \pm 0.01, \pm 0.001$ and estimate the limit at 0.
- (b) For $f(x) = x^4$, use $D(h) = [f(1+h) - f(1)]/h$ at the same six steps; estimate $f'(1)$.
- (c) Plot $(e^x - 1)/x$ near 0. Compare with $|x - 1|/(x - 1)$ at $x = 0.9, 0.99, 1.01, 1.1$. Explain the different behaviour.
- (d) Try $h = 10^{-12}$ in the difference quotient. Explain why smaller is not automatically more reliable, even if your calculator gives a plausible result.

10. Numerical derivatives, tangents and normals

Syllabus: 5.3–5.7, 5.15

Calculator method

Use **Scratchpad Calculate** → **Menu** → **Calculus** → **Numerical Derivative at a Point**, or the Catalog entry **nDerivative**. Example: **nDerivative(f1(t),t=0.7,2)** evaluates the second derivative. For graphs enter **nDerivative(f1(t),t=x)** and order 2 in separate function lines. Obtain $m = f'(a)$, then enter $y = f(a) + m(x - a)$ and $y = f(a) - (x - a)/m$ when $m \neq 0$. Cross-check a first derivative with **Scratchpad Graph** → **Menu** → **Analyze Graph** → **dy/dx** at the chosen input.

19. Let $f(x) = e^x \sin x$ and $a = 0.7$.

- (a) Calculate $f(a)$, $f'(a)$ and $f''(a)$ numerically, storing the unrounded values.
- (b) Plot f , its numerical first derivative and its numerical second derivative on $-2 \leq x \leq 2$. Evaluate both derivative graphs at a as a check.
- (c) Use **Analyze Graph** → **dy/dx** at a to check the slope. Write the tangent and normal equations at a , using stored coefficients. Plot them with f and zoom in on the contact point.
- (d) Solve $f'(x) = 2$ on $0 \leq x \leq 1$ using the derivative graph. Why does a derivative value not provide an algebraic formula for $f'(x)$?

20. Let $g(x) = \ln(1 + x^2)$ and $a = 1.2$.

- (a) Find and store $g(a)$, $g'(a)$ and $g''(a)$ numerically.
- (b) Plot g , its numerical first derivative and its numerical second derivative on $-3 \leq x \leq 3$. Check the values at a .
- (c) Use **Analyze Graph** → **dy/dx** at a to check the slope. Find and graph the tangent and normal at a . Zoom in to check their contact and directions.
- (d) Solve $g'(x) = 0.8$ for $x > 0$, using a window that shows every solution. Find the tangent and normal at $x = 0$, where the usual reciprocal-gradient formula needs special treatment.



11. Stationary points, inflexions and global extrema

Syllabus: 5.2, 5.7–5.8, 5.14

Calculator method

Find zeros of a numerical derivative graph, then evaluate the original function. Check the sign of f' on both sides to classify stationary points. A zero of f'' is only a candidate inflexion: check concavity changes. For a global optimum on a closed interval, compare all stationary points and endpoints. A calculator's Maximum command searches the selected region.

21. Let $f(x) = xe^{-x/2}$ on $0 \leq x \leq 6$.
- Graph f, f', f'' using numerical derivatives. Find and classify all stationary points using signs of f' on either side.
 - State the increasing and decreasing intervals. Find the inflexion point; verify a sign change in f'' .
 - State the concave-up and concave-down intervals.
 - Compare the stationary value with $f(0)$ and $f(6)$. Find the absolute maximum and minimum on the domain.
 - Investigate $u(x) = x^4$ near 0. Use $u''(0)$ and values on both sides to explain why $u''(0) = 0$ does not guarantee an inflexion.
22. Let $g(x) = x^3 - 6x^2 + 9x + 1$ on $0 \leq x \leq 5$.
- Plot g, g', g'' and find all stationary points. Classify them using derivative signs.
 - State increasing/decreasing intervals and find the inflexion point, checking the second derivative on both sides.
 - State the concavity intervals.
 - Compare the values at all stationary points and both endpoints. State the absolute maximum and minimum, including all inputs where each occurs.
 - Investigate $u(x) = (x - 1)^4$ near 1. Explain why the zero of u'' is not an inflexion point.

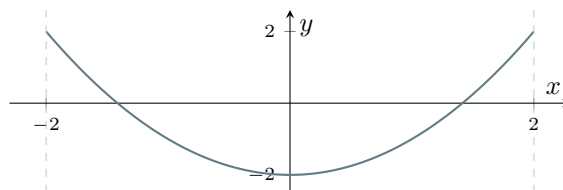
12. Definite integrals and total area

Syllabus: 5.5, 5.10–5.11

Calculator method

Use the definite-integral template in **Scratchpad Calculate** → **Menu** → **Calculus**; enter both limits, integrand and variable. In Scratchpad Graph use **Analyze Graph** → **Integral** with the same bounds to inspect the signed contribution. For total area enter $|f(x)|$ (abs) or split at every zero and make each contribution positive. Always write the integral before evaluating it.

23. Let $f(x) = x^2 - 2$ on $[-2, 2]$.
- Graph f and find its zeros. Use the definite-integral template to calculate $I = \int_{-2}^2 f(x) dx$.
 - Use Analyze Graph → Integral to reproduce the result on the graph. Explain why it is negative.
 - Calculate the total area between the curve and the x -axis using $\int_{-2}^2 |f(x)| dx$.
 - Check by splitting the integral at the zeros. State why $|I|$ is not the required area.





24. Let $g(x) = \sin x - 0.25$ on $[0, 2\pi]$.
- (a) Find its zeros and store them without rounding. Calculate $\int_0^{2\pi} g(x) dx$ with the template.
 - (b) Reproduce the signed result with the Graphs Integral command.
 - (c) Set up and evaluate the total area using **abs**. Check by splitting at both zeros and reversing the signs of negative contributions.
 - (d) Compare total area with the absolute value of the signed integral, and explain the difference.

13. Area between curves and integration in y

Syllabus: 5.11, 5.17

Calculator method

Locate every boundary intersection before integrating. Integrate upper minus lower with respect to x , or right minus left with respect to y . Use absolute difference if curve order changes. For a relation $x = u(y)$, graphing $u(x)$ uses the screen's horizontal variable as a stand-in for y ; it does not preserve the original orientation. Use a labelled sketch to show the actual region.

25. Find areas in two orientations.
- (a) Graph $y = x^3$ and $y = 3x$. Find all three intersections and store their x -coordinates.
 - (b) Sketch the two bounded regions. Set up and calculate their combined area; verify by splitting where the upper curve changes.
 - (c) A different region lies between $x = y^2$ and $x = y + 2$. Solve for its boundary y -values using the calculator.
 - (d) Sketch this region with x horizontal and y vertical. Identify the right-hand curve, then evaluate its area with respect to y .
26. Repeat with another pair of regions.
- (a) Graph $y = x^3$ and $y = 4x$. Find every intersection, retaining the boundary values.
 - (b) Set up and evaluate the total area of both bounded regions using absolute difference; check by splitting at the change of order.
 - (c) Find the boundary y -values for the region between $x = y^2$ and $x = 2y + 3$.
 - (d) Sketch the region in the correct orientation. Set up and evaluate an integral with respect to y .

14. Unknown bounds and parameters inside integrals

Syllabus: 2.10; 5.5, 5.11

Calculator method

Use a dummy integration variable, such as t , distinct from the unknown upper bound. To solve $\int_0^b f(t) dt = c$, graph the integral with upper limit x and the constant c , or use **nSolve** with a sensible guess and bounds. Integral-based graphing can be slow. Substitute the unrounded solution back into the integral to verify it.

27. Define $F(b) = \int_0^b e^{-t^2} dt$, $0 \leq b \leq 2$.
- (a) Graph $F(x)$ and $y = 0.6$ to estimate the required upper bound.
 - (b) Solve $F(b) = 0.6$ numerically and store b . Verify the original integral using that stored value.
 - (c) Find the positive constant k for which $\int_0^1 ke^{-t^2} dt = 2$. Show the numerical equation before solving it.
 - (d) Explain why the bound in part (b) is unique. Why is t a safer integration variable than b in that calculation?



28. Define $G(b) = \int_0^b \frac{1}{1+t^3} dt$, $0 \leq b \leq 2$.

- (a) Graph $G(x)$ together with $y = 0.7$.
- (b) Solve $G(b) = 0.7$ with a suitable guess and bounds; store b and verify the integral.
- (c) Find $k > 0$ such that $\int_0^1 \frac{k}{1+t^3} dt = 3$. Show your equation and check the result.
- (d) Explain uniqueness of the bound and why retaining full precision matters when checking it.

15. Kinematics: direction, speed and distance

Syllabus: 5.9

Calculator method

For a velocity function v , use zeros for possible direction changes, $|v|$ for speed and a numerical derivative for acceleration. Displacement is $\int v dt$; total distance is $\int |v| dt$. Recover position as $s(0) + \int_0^t v(u) du$. An object speeds up when v and a have the same sign, and slows down when their signs differ.

29. A particle has velocity $v(t) = t^2 - 4t + 3 \text{ m s}^{-1}$ for $0 \leq t \leq 4$, with $s(0) = 2 \text{ m}$.

- (a) Graph v and $|v|$. Find the times at rest and state when it moves in each direction.
- (b) Find velocity, speed and acceleration at $t = 0.5$. Is it speeding up or slowing down?
- (c) Write and calculate the displacement and total distance over the four seconds.
- (d) Define the position using an integral with dummy variable u . Find $s(4)$ and the maximum and minimum positions on $[0, 4]$.
- (e) Find when the particle is speeding up, using velocity and acceleration graphs.

30. A second particle has velocity $v(t) = \cos t - 0.3 \text{ m s}^{-1}$ for $0 \leq t \leq 2\pi$, with $s(0) = -1 \text{ m}$.

- (a) Graph velocity and speed; find both rest times and the direction intervals.
- (b) Find velocity, speed and acceleration at $t = 0.5$ and say whether speed is increasing.
- (c) Set up and calculate displacement and total distance on the full interval.
- (d) Define position with a numerical integral. Find $s(2\pi)$ and its absolute maximum and minimum, checking endpoints.
- (e) Use the signs of v and a to find the speeding-up intervals.

16. Volumes about either axis

Syllabus: 5.17

Calculator method

Draw the region and identify the radius perpendicular to the rotation axis. About the x -axis use $\pi \int y^2 dx$; about the y -axis use $\pi \int x^2 dy$. For a hollow cross-section use $\pi \int (R^2 - r^2)$, never $\pi \int (R - r)^2$. Put the entire radius expression in brackets before squaring and check the integration variable and limits.

31. All lengths below are in centimetres. Sketch each region and mark its rotation axis.

- (a) Rotate the region under $y = e^{-x^2}$, above the x -axis, from $x = 0$ to 1, about the x -axis. Set up and evaluate its volume.
- (b) Rotate the region between $x = 0$ and $x = \ln(y + 1)$, $0 \leq y \leq 2$, about the y -axis. Set up and evaluate its volume using y .
- (c) On $0 \leq x \leq 1$, the region between $y = 2 - x$ and $y = x$ is rotated about the x -axis. Identify both radii and calculate the volume.
- (d) Explain why the square of the difference of the two radii would give the wrong result.



32. Repeat for the following regions, with lengths in centimetres.

- (a) Rotate the region under $y = \ln(x + 1)$ and above the x -axis, $0 \leq x \leq 2$, about the x -axis.
- (b) Rotate the region between $x = 0$ and $x = e^{-y^2}$, $0 \leq y \leq 1$, about the y -axis.
- (c) Rotate the region between $x = 1$ and $x = 3 - y$, $0 \leq y \leq 2$, about the y -axis. Show the outer and inner radii on a sketch.
- (d) For each volume, write the integral before calculating it. State the units and distinguish radius from diameter.

17. Euler's method in a spreadsheet

Syllabus: 5.18

Calculator method

For $y' = F(x, y)$, use $x_{n+1} = x_n + h$, $y_{n+1} = y_n + hF(x_n, y_n)$. For the spreadsheet steps only, choose **Home** → **New Document** → **Add Lists & Spreadsheet**. Put x_0, y_0 in cells A1,B1. Enter $=a1+0.2$ in A2 and the appropriate update in B2; use **Data** → **Fill Down** on A2:A4 and B2:B4. Use old-row values in both updates. Return to **Scratchpad Calculate** to cross-check with `euler(F(x,y),x,y,{0,0.6},y0,0.2,1)`; the final 1 prevents hidden substeps. Re-enter expressions there: spreadsheet-document variables are separate from Scratchpad variables.

33. Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$, approximately to $x = 0.6$.

- (a) With $h = 0.2$, show the first update by hand, then enter B2 as $=b1+0.2*(a1+b1)$ and fill down. Copy all four (x, y) rows.
- (b) Verify with `euler` in Scratchpad Calculate. Return to your spreadsheet document and repeat the calculation with $h = 0.1$; fill to row 7.
- (c) In Scratchpad Calculate, compare both recorded estimates with $y(x) = 2e^x - x - 1$. Calculate the absolute errors at 0.6.
- (d) Switch to **Scratchpad Graph: Graph Entry/Edit** → **Diff Eq**; enter $x+y1$ and initial condition $(0, 1)$. Select Euler, Plot Step 0.2, Iterations Between Plot Step 1, Field Slope; plot from 0 to 0.6. Compare the plotted solution with the table.

34. Solve $\frac{dy}{dx} = 2x - y$, $y(0) = 2$, approximately to $x = 0.6$.

- (a) In your spreadsheet document, add a fresh **Lists & Spreadsheet** page. For $h = 0.2$, enter B2 as $=b1+0.2*(2*a1-b1)$. Show one update and copy all rows up to $x = 0.6$.
- (b) Check with `euler` in Scratchpad Calculate; return to the spreadsheet and repeat with $h = 0.1$ and record $y(0.6)$.
- (c) In Scratchpad Calculate, compare with $y(x) = 2x - 2 + 4e^{-x}$ and calculate the two absolute errors.
- (d) In **Scratchpad Graph**, graph the differential equation using $2*x-y1$, initial condition $(0, 2)$, and the same Euler/step/iteration settings as above. Inspect the slope field and compare the curve with your table. Explain why Runge-Kutta would not reproduce the requested Euler calculation.



18. Checking analytical work and related rates

Syllabus: 5.10, 5.12, 5.14–5.16

Calculator method

The non-CAS CX evaluates derivatives and definite integrals numerically; it does not supply general symbolic antiderivatives, implicit derivatives or series proofs. Derive the required expressions by hand, then check them numerically at valid points. For implicit curves, an explicit branch can provide a numerical derivative check. For a related rate, establish the relationship before substituting values.

35. Use the calculator to verify analytical work.

- (a) For $x^2 + xy + y^2 = 7$, derive dy/dx . Evaluate it at $(1, 2)$. Graph the positive branch $y = (-x + \sqrt{28 - 3x^2})/2$ and check the slope numerically at $x = 1$.
- (b) A sphere's radius increases at 0.04 cm s^{-1} . Use $V = 4\pi r^3/3$ to find dV/dt when $r = 3 \text{ cm}$. Verify dV/dr numerically before multiplying by dr/dt .
- (c) Check that $F(x) = e^x(x-1)$ is an antiderivative of xe^x at $x = 0.3$ and 0.8 . Compare $F(1) - F(0)$ with $\int_0^1 xe^x dx$.
- (d) Check the substitution result $\int_0^1 \frac{2x}{1+x^2} dx = \ln 2$ numerically.
- (e) Derive the third derivative of $p(x) = x^4 - 2x^3$ by hand and use the calculator to evaluate it at $x = 0.7$. Explain why this is not a symbolic third-derivative command.

36. Repeat these checks with different expressions.

- (a) For $x^2 + 2y^2 = 9$, derive dy/dx and evaluate it at $(1, 2)$. Check against the numerical derivative of $y = \sqrt{(9 - x^2)/2}$ at $x = 1$.
- (b) A cube's edge increases at 0.05 cm s^{-1} . Using $V = s^3$, find dV/dt at $s = 4 \text{ cm}$; verify dV/ds numerically.
- (c) Check the proposed antiderivative $F(x) = x \sin x + \cos x$ of $x \cos x$ at 0.3 and 0.8 . Compare $F(1) - F(0)$ with its definite integral.
- (d) Check the partial-fraction result $\int_0^1 \frac{1}{(x+1)(x+2)} dx = \ln(4/3)$.
- (e) Derive the third derivative of $p(x) = x^5 - x^3$ by hand; evaluate it at $x = 0.7$. Explain why numerical spot-checks do not prove an identity.

19. Maclaurin approximations and error intervals

Syllabus: 5.13, 5.19

Calculator method

Enter the original function and the supplied polynomial separately. Evaluate absolute error $|f(x) - P(x)|$ and percentage relative error $100|f(x) - P(x)|/|f(x)|$ when $f(x) \neq 0$. Graph the error with a horizontal tolerance line, then solve for the boundary and check which side satisfies it. Avoid repeatedly nesting numerical derivatives to manufacture high-order coefficients.

37. For $f(x) = e^x$, use $P_3(x) = 1 + x + x^2/2 + x^3/6$.

- (a) Define and graph f and P_3 in a window around 0. Evaluate both at $x = 0.4$.
- (b) Calculate the absolute error and percentage relative error at 0.4 , retaining full precision.
- (c) On $0 \leq x \leq 1$, graph $E(x) = |e^x - P_3(x)|$ and $y = 0.001$. Find the positive intersection and determine the interval on which $E(x) < 0.001$.
- (d) Test inputs on both sides of the boundary. State the interval using an unrounded stored endpoint before giving its decimal approximation.
- (e) Use a two-sided table to estimate $\lim_{x \rightarrow 0} \frac{e^x - P_3(x)}{x^4}$; use $x = \pm 0.1, \pm 0.05, \pm 0.02$. Check using the next series term.



38. For $f(x) = \ln(1+x)$, use $P_3(x) = x - x^2/2 + x^3/3$.
- (a) Graph both functions around 0, respecting $x > -1$. Evaluate them at $x = 0.3$.
 - (b) Find the absolute error and percentage relative error at 0.3.
 - (c) On $0 \leq x \leq 0.8$, graph the absolute error and the tolerance 0.001. Find the positive boundary and state the interval where the error is strictly below it.
 - (d) Verify with test values on both sides. Explain why percentage relative error is undefined at $x = 0$ here, although absolute error is zero.
 - (e) Estimate $\lim_{x \rightarrow 0} \frac{\ln(1+x) - P_3(x)}{x^4}$ using $x = \pm 0.1, \pm 0.05, \pm 0.02$; compare with the next series term.

20. Integrated calculator practice: exam style

Syllabus: 2.3–2.4, 2.10, 2.15; 5.4, 5.8, 5.11

Calculator method

For these original calculator-allowed questions, choose the methods yourself. Show equations, relevant coordinates, integral setups and contextual units; a calculator command alone is not a complete mathematical solution. Record enough intermediate values to make the reasoning visible, and round only the final answers. These use IB-style command terms and mark allocations.

39. [Maximum mark: 15] The curves $y = e^{-x}$ and $y = 0.3x$ meet at P , where $x > 0$. The region R is bounded by these curves and the y -axis.
- (a) Find the coordinates of P . [3]
 - (b) Sketch both curves and shade R . Hence find the area of R . [4]
 - (c) Find the equation of the tangent to $y = e^{-x}$ at P . [3]
 - (d) The line $x = c$ divides R into two equal areas. Find c . [3]
 - (e) State the solution of $e^{-x} > 0.3x$ for $x \geq 0$, referring to the stored intersection value. [2]

Record the bounds/guess used and the integral used for part (d).

40. [Maximum mark: 15] The curves $y = \ln(x+1)$ and $y = 0.5x$ intersect at $O = (0, 0)$ and at a second point P . They enclose a region R in the first quadrant.
- (a) Find the coordinates of P and sketch R . [3]
 - (b) Find the area of R . [3]
 - (c) Find the maximum vertical separation of the two curves within R , and the input where it occurs. [3]
 - (d) Find the equation of the tangent to $y = \ln(x+1)$ at P . [3]
 - (e) Find c such that $x = c$ divides R into two equal areas. [3]

Explain how you prevented **nSolve** from returning $x = 0$ when finding P .



Answer key and solution outlines

Values below retain extra digits so that calculations can be checked. Student final answers normally use three significant figures. A screen plot supports a sketch; it does not replace labels or written reasoning.

1. Set up, define and evaluate

1. (a) -1.085714286 , -10 . (b) $f_1(\sqrt{2}) = 1.656854249 \approx 1.66$. (c) $f_1(\sqrt{2}) - f_1(1.41) \approx 0.0119165$. Storing keeps the calculator's internal precision. (d) $x \neq -1$; Scratchpad Calculate and Graph share the same definition. (e) $0, 0, 1$; an open point at $(0, 1)$ and a filled point at $(0, 0)$. Extra entry checks: $\log_2 7 \approx 2.807354922$, $\arcsin(0.5) = \pi/6 \approx 0.523599$ radians.
2. (a) 1.504323081 , $1 + \ln 2 \approx 1.693147181$. (b) $1.493879103 \approx 1.49$. (c) $f_1(\sqrt{3}) - f_1(1.73) \approx 0.000186460$. (d) $x > -2$. After switching back, $f_1(0.6) \approx 1.504323081$ is still available in Scratchpad Calculate. (e) $2, 1, 4$; the two pieces meet at $(1, 1)$. Extra entry checks: $\log_3 10 \approx 2.095903274$, $\arccos(0.5) = \pi/3 \approx 1.047198$ radians.

2. Windows, traces and tables

3. (a) The vertex at $(20, -3)$ is outside it. (b) -2.875 . (c) For $x = 15, 17.5, 20, 22.5, 25$, $f_1 = -2.5, -2.875, -3, -2.875, -2.5$; $f_2 = -0.5, -0.25, 0, 0.25, 0.5$. (d) $f_1(18.3) = -2.9422$; sum and difference at 20 are both -3 .
4. (a) -0.485918166 . (b) An excessively wide vertical range hides small oscillations. (c) $f_1 = 1, 0.442362, -0.278952, -0.543319, -0.293701$; $f_2 = 0.2$ throughout. (d) $f_1(1.3) \approx -0.509437971$; sum 1.2, difference 0.8.

3. Restrictions, intercepts and extrema

5. Zeros: $-\sqrt{3}, 0, \sqrt{3} \approx -1.732, 0, 1.732$. Local maximum $(-1, 2)$; local minimum $(1, -2)$. Endpoints $(-2, -2)$ and $(2.5, 8.125)$. Absolute minimum -2 at $x = -2, 1$; absolute maximum 8.125 at 2.5 . Range $[-2, 8.125]$.
6. Zeros $\pm\sqrt{2 \pm \sqrt{3}} \approx \pm 0.517638, \pm 1.931852$; y -intercept $(0, 1)$. Local minima $(\pm\sqrt{2}, -3)$; local maximum $(0, 1)$. Endpoint values $1, 5.0656$. Range $[-3, 5.0656]$; absolute maximum at 2.2 . The right endpoint is higher than the local maximum.

4. All intersections, numerical roots and inequalities

7. Roots $r_1 = -0.458962268$, $r_2 = 0.910007572$, $r_3 = 3.733079029$. Intersection heights are $3r_i^2 \approx 0.631939, 2.48434, 41.8076$. Suitable brackets: $[-1, 0]$, $[0, 1]$, $[3, 4]$. Difference signs are $-, +, -, +$. Solution: $[r_1, r_2] \cup [r_3, 5]$. Residuals should be close to zero; three roots exist.
8. Domain $x > -3$. Roots $r_1 = -0.869647810$, $r_2 = 1.197723858$; heights $\approx 0.756287, 1.43454$. Brackets $[-1, 0]$, $[1, 2]$. The difference is positive only on (r_1, r_2) . At -3 the logarithm is undefined; quoted decimals approximate the boundary roots.



5. Polynomial roots and linear systems

9. (a) $\{1, 0, -5, 0, 4\}$ gives $-2, -1, 1, 2$. (b) $[-2, -1] \cup [1, 2]$. (c) $c = 2$, $a + b + c = 0$, $9a + 3b + c = 2$, giving $(a, b, c) = (1, -3, 2)$. (d) $p = x^2 - 3x + 2$; vertex $(1.5, -0.25)$. Complex-root check: $x^2 + 1 = 0$ has no real roots (empty list) and complex roots $\pm i$.

10. (a) $\{1, -2, -5, 6\}$ gives $-2, 1, 3$. (b) $(-2, 1) \cup (3, \infty)$. (c) $a - b + c = 6$, $a + b + c = 2$, $4a + 2b + c = 3$, so $(a, b, c) = (1, -2, 3)$. (d) $q = (x - 1)^2 + 2$; vertex $(1, 2)$. Complex-root check: $x^2 + 4 = 0$ has no real roots (empty list) and complex roots $\pm 2i$.

6. Asymptotes, exclusions and misleading screens

11. (a) Values $-498, 502, 1.950980, 2.051020$; intercepts $(-0.5, 0), (0, -0.5)$; asymptotes $x = 2$, $y = 2$. (b) $g = x + 1 + 2/(x - 1)$, differences $-2/101, 2/99$; asymptotes $x = 1$, $y = x + 1$; only real intercept $(0, -1)$. (c) $h = x + 1$ for $x \neq 1$: 1.99, undefined, 2.01. Hole $(1, 2)$; nearby values stay finite.

12. (a) Values $503, -497, 3.050505, 2.950495$; intercepts $(2/3, 0), (0, -2)$; asymptotes $x = -1$, $y = 3$. (b) $g = x - 2 + 8/(x + 2)$; differences $-8/98, 8/102$; asymptotes $x = -2$, $y = x - 2$; only real intercept $(0, 2)$. (c) $h = x - 2$ for $x \neq -2$; values -4.01 , undefined, -3.99 ; hole $(-2, -4)$.

7. Composition, inverse, symmetry and period

13. (a) 48, 9; $f \circ g$ needs $x \geq -3/2$, $g \circ f$ needs $x \geq 0$. (b) $f^{-1}(x) = \sqrt{x + 1}$: $f: [0, \infty) \rightarrow [-1, \infty)$ and inverse $[-1, \infty) \rightarrow [0, \infty)$. (c) 2, 3; inverse reverses inputs and outputs. (d) Both values -0.7399 , and $p(-x) = p(x)$: even. (e) $T = \pi$.

14. (a) 9, 1; both compositions have domain $x \geq 1$. (b) $f^{-1}(x) = 1 + \sqrt{x}$, with domain $[0, \infty)$ and range $[1, \infty)$; these reverse those of f . (c) 3, 4; $f^{-1}(4) = 3$ but $1/f(4) = 1/9$. (d) $p(0.7) = -0.357$, $p(-0.7) = 0.357$; odd. (e) $T = 2\pi/3$.

8. Transformations, modulus and parameters

15. (a) $(3, 9)$ and $(0.5, 3)$. (b) At -2 : 3, 3; at 0: 1, -1 . Output modulus reflects negative heights; input modulus uses the right half and reflects it left. (c) Reciprocal has no zeros, excludes ± 1 ; square has zeros ± 1 , domain $\mathbb{R}$. (d) Roots $\pm\sqrt{0.5}, \pm\sqrt{1.5}$; strict inequality on $(-\sqrt{1.5}, -\sqrt{0.5}) \cup (\sqrt{0.5}, \sqrt{1.5})$. (e) $k = 1$.

16. (a) $(0, -6)$ and $(4/3, 7)$. (b) At -2 : 9, 7; at 0: 1, -1 . (c) Reciprocal excludes 1, has no zero; square has zero 1, domain $\mathbb{R}$. (d) Roots $-1, \sqrt[3]{3}$; solution $[-1, \sqrt[3]{3}]$. (e) For $k = -1, 0, 1$: zero, one, two roots respectively; threshold $k = 0$.

**9. Limits and difference quotients**

- 17.** (a) For either sign: 0.998334166, 0.999983333, 0.999999833; limit 1, but the expression is undefined at 0. (b) $D(h) = 12 + 6h + h^2$: at positive steps 12.61, 12.0601, 12.006001; negative steps 11.41, 11.9401, 11.994001; limit 12. (c) $|x|/x$ has one-sided limits -1 and 1 , so no two-sided limit. (d) A graph can conceal an undefined point.
- 18.** (a) Positive steps: 1.051709181, 1.005016708, 1.000500167; negative: 0.951625820, 0.995016625, 0.999500167; limit 1. (b) $D(h) = 4 + 6h + 4h^2 + h^3$ tends to 4; values 4.641, 4.060401, 4.006004001 and 3.439, 3.940399, 3.994003999. (c) The second expression tends to -1 from the left and 1 from the right at 1. (d) Subtraction of nearly equal numbers loses useful digits.

10. Numerical derivatives, tangents and normals

- 19.** $f(0.7) = 1.297295112$, $m = 2.837498137$, $f''(0.7) = 3.080406051$. Tangent $y - 1.297295112 = 2.837498137(x - 0.7)$; normal $y - 1.297295112 = -(x - 0.7)/2.837498137$. For the solution graph, analytical checks are $f' = e^x(\sin x + \cos x)$, $f'' = 2e^x \cos x$. $f'(x) = 2$ gives $x \approx 0.416296802$. One evaluation gives a number at one input only.
- 20.** $g(1.2) = 0.891998039$, $m = 0.983606557$, $g''(1.2) = -0.147809729$. Tangent $y - 0.891998039 = 0.983606557(x - 1.2)$; normal gradient -1.016666667 . Derivative checks: $g' = 2x/(1 + x^2)$, $g'' = 2(1 - x^2)/(1 + x^2)^2$. Slopes 0.8 at $x = 0.5, 2$. At 0: tangent $y = 0$, normal $x = 0$.

11. Stationary points, inflexions and global extrema

- 21.** $f' = e^{-x/2}(1 - x/2)$, so the stationary point $(2, 2/e) \approx (2, 0.735759)$ is a maximum. Increasing on $(0, 2)$, decreasing on $(2, 6)$. $f'' = e^{-x/2}(x/4 - 1)$ changes from $-$ to $+$ at 4; inflexion $(4, 4e^{-2}) \approx (4, 0.541341)$. Concave down $(0, 4)$, up $(4, 6)$. Endpoints: 0, $6e^{-3} \approx 0.298722$; global minimum 0 at 0. For u , $u'' = 12x^2 > 0$ on both sides: no inflexion.
- 22.** $g' = 3(x - 1)(x - 3)$: signs $+, -, +$, hence maximum $(1, 5)$ and minimum $(3, 1)$. Increasing $(0, 1) \cup (3, 5)$, decreasing $(1, 3)$. $g'' = 6x - 12$: concave down $(0, 2)$, up $(2, 5)$; inflexion $(2, 3)$. Candidate values at 0, 1, 3, 5: 1, 5, 1, 21. Global maximum 21 at 5; minimum 1 at 0, 3. $u'' = 12(x - 1)^2$ has no sign change.

12. Definite integrals and total area

- 23.** Zeros $\pm\sqrt{2}$. $I = -8/3 \approx -2.66667$ because the below-axis contribution dominates. Total area $A = (16\sqrt{2} - 8)/3 \approx 4.87581$. Split as $\int_{-2}^{-\sqrt{2}} f - \int_{-\sqrt{2}}^{\sqrt{2}} f + \int_{\sqrt{2}}^2 f$. Taking $|I|$ retains cancellation instead of adding all geometric areas.
- 24.** Zeros $a = \arcsin(1/4) \approx 0.252680255$, $b = \pi - a \approx 2.888912398$. Signed integral $-\pi/2 \approx -1.57080$. Area $-\int_0^a g + \int_a^b g - \int_b^{2\pi} g \approx 4.12566$. Equivalently $A = \sqrt{15} + \arcsin(1/4)$. The integral subtracts negative regions; total area adds their magnitudes.

**13. Area between curves and integration in y**

25. (a) $x = -\sqrt{3}, 0, \sqrt{3}$; coordinates $(x, 3x)$. (b) $A = \int_{-\sqrt{3}}^0 (x^3 - 3x) dx + \int_0^{\sqrt{3}} (3x - x^3) dx = 9/2$. Each region has area $9/4$. (c) $y = -1, 2$. (d) Right boundary $x = y + 2$: $A = \int_{-1}^2 (y + 2 - y^2) dy = 9/2$.

26. (a) $(-2, -8), (0, 0), (2, 8)$. (b) $A = \int_{-2}^0 (x^3 - 4x) dx + \int_0^2 (4x - x^3) dx = 8$. (c) $y = -1, 3$. (d) Right boundary $x = 2y + 3$: $A = \int_{-1}^3 (2y + 3 - y^2) dy = 32/3 \approx 10.6667$.

14. Unknown bounds and parameters inside integrals

27. $b \approx 0.698881648$ (for example, guess 0.7, bounds 0, 2), giving 0.600000. Since $\int_0^1 e^{-t^2} dt \approx 0.746824133$, $0.746824133k = 2$, so $k \approx 2.678006658$. The integrand is positive, so F increases strictly. A separate dummy variable distinguishes the integration process from the unknown bound.

28. $b \approx 0.770583787$; substitution gives 0.700000. $\int_0^1 (1 + t^3)^{-1} dt \approx 0.835648848$; thus $k \approx 3.590024693$. The integrand is positive on $[0, 2]$, so G increases strictly. Rounded bounds can introduce avoidable error in the verification.

15. Kinematics: direction, speed and distance

29. Rest at $t = 1, 3$; $v > 0$ on $[0, 1) \cup (3, 4]$, $v < 0$ on $(1, 3)$. At 0.5: $v = 1.25$, speed 1.25 m s^{-1} , $a = -3 \text{ m s}^{-2}$, slowing. Displacement $\int_0^4 v = 4/3 \text{ m}$; distance $\int_0^4 |v| = 4 \text{ m}$. $s(t) = 2 + t^3/3 - 2t^2 + 3t$; $s(4) = 10/3 \text{ m}$. Maximum $10/3 \text{ m}$ at $t = 1, 4$; minimum 2 m at $t = 0, 3$. Speeding up on $(1, 2) \cup (3, 4)$.

30. Let $a_0 = \arccos 0.3 \approx 1.266103673$, $b_0 = 2\pi - a_0 \approx 5.017081634$. Positive velocity before a_0 and after b_0 , negative between. At 0.5: $v = |v| \approx 0.577583 \text{ m s}^{-1}$, $a \approx -0.479426 \text{ m s}^{-2}$, slowing. Displacement $-0.6\pi \approx -1.884956 \text{ m}$; distance $\approx 4.181388 \text{ m}$. $s = -1 + \sin t - 0.3t$; $s(2\pi) \approx -2.884956 \text{ m}$. Maximum $s(a_0) \approx -0.425892 \text{ m}$; minimum $s(b_0) \approx -3.459063692 \text{ m}$. Speeding up on $(a_0, \pi) \cup (b_0, 2\pi)$.

16. Volumes about either axis

31. (a) $V = \pi \int_0^1 (e^{-x^2})^2 dx \approx 1.879124817 \text{ cm}^3$. (b) $V = \pi \int_0^2 [\ln(y+1)]^2 dy \approx 3.233242809 \text{ cm}^3$. (c) $R = 2 - x$, $r = x$; $V = \pi \int_0^1 [(2-x)^2 - x^2] dx = 2\pi \approx 6.28319 \text{ cm}^3$. (d) Cross-sectional area is outer disk area minus inner disk area.

32. (a) $\pi \int_0^2 [\ln(x+1)]^2 dx \approx 3.233242809 \text{ cm}^3$. (b) $\pi \int_0^1 (e^{-y^2})^2 dy \approx 1.879124817 \text{ cm}^3$. (c) $R = 3 - y$, $r = 1$; $V = \pi \int_0^2 [(3-y)^2 - 1] dy = 20\pi/3 \approx 20.943951 \text{ cm}^3$. Every radius is the perpendicular distance from the axis; a diameter is twice that distance.

**17. Euler's method in a spreadsheet**

33. For $h = 0.2$: $(0, 1), (0.2, 1.2), (0.4, 1.48), (0.6, 1.856)$. For $h = 0.1$: $1, 1.1, 1.22, 1.362, 1.5282, 1.72102, 1.943122$. Exact $y(0.6) \approx 2.044237601$; errors $0.188237601, 0.101115601$. `euler(x+y,x,y,{0,0.6},1,0.2,1)` returns the same coarse table. The solution is increasing; the smaller step improves this estimate.

34. For $h = 0.2$: $(0, 2), (0.2, 1.6), (0.4, 1.36), (0.6, 1.248)$. For $h = 0.1$: $2, 1.8, 1.64, 1.516, 1.4244, 1.36196, 1.325764$. Exact $y(0.6) \approx 1.395246544$; errors $0.147246544, 0.069482544$. Use `euler(2*x-y,x,y,{0,0.6},2,0.2,1)`. Runge-Kutta is a different numerical algorithm; matching output spacing alone does not make it Euler's method.

18. Checking analytical work and related rates

35. (a) $dy/dx = -(2x + y)/(x + 2y)$; at $(1, 2)$ it is $-4/5 = -0.8$. (b) $dV/dr = 4\pi r^2 = 36\pi \text{ cm}^2$; $dV/dt = 1.44\pi \approx 4.52389 \text{ cm}^3 \text{ s}^{-1}$. (c) $F' = xe^x$; the two checks give $0.404958, 1.780433$, and both definite evaluations give 1. (d) 0.693147181 . (e) $p^{(3)} = 24x - 12$; value 4.8. The symbolic differentiation was done by hand.

36. (a) $dy/dx = -x/(2y)$; value $-1/4 = -0.25$. (b) $dV/ds = 3s^2 = 48 \text{ cm}^2$; $dV/dt = 2.4 \text{ cm}^3 \text{ s}^{-1}$. (c) $F' = x \cos x$; values $0.286601, 0.557365$; integral $\sin 1 + \cos 1 - 1 \approx 0.381773291$. (d) $\ln(4/3) \approx 0.287682072$. (e) $p^{(3)} = 60x^2 - 6$; value 23.4. Agreement at finitely many inputs does not establish equality everywhere.

19. Maclaurin approximations and error intervals

37. At 0.4: $f = 1.491824698$, $P_3 = 1.490666667$; absolute error 0.001158031 ; percentage error 0.0776251% . Let r be the stored root of $E(x) = 0.001$: $r \approx 0.385880785$; the interval is $[0, r)$. For example 0.3 satisfies the tolerance, 0.4 does not. The error increases on the positive interval. The signed quotient tends to $1/24 \approx 0.0416667$, the coefficient of x^4 .

38. At 0.3: $f = 0.262364264$, $P_3 = 0.264$; absolute error 0.001635736 ; percentage error 0.623460% . Boundary $r \approx 0.263728102$; interval $[0, r)$. The value 0.2 meets the tolerance, 0.3 does not. Relative error divides by $|\ln 1| = 0$. The signed quotient tends to $-1/4$, consistent with the next term $-x^4/4$.

20. Integrated calculator practice: exam style

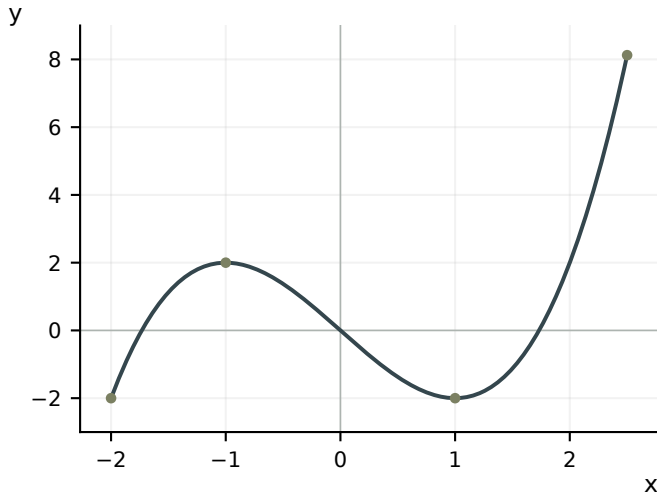
39. (a) Solve $e^{-x} = 0.3x$ with a positive bracket, giving $p \approx 1.104542018$; $P \approx (1.10454, 0.331363)$. (b) $A = \int_0^p (e^{-x} - 0.3x) dx \approx 0.485635434$. (c) $m = -e^{-p} \approx -0.331362605$; $y - 0.331362605 = -0.331362605(x - 1.104542018)$. (d) Solve $\int_0^c (e^{-x} - 0.3x) dx = A/2$, with $0 < c < p$; $c \approx 0.295614683$. (e) $0 \leq x < p$.

40. (a) Solve $\ln(x+1) = 0.5x$ with bounds 2, 3 and guess 2.5: $p \approx 2.512862417$, $P \approx (2.51286, 1.25643)$. (b) $A = \int_0^p [\ln(x+1) - 0.5x] dx \approx 0.322188173$. (c) Maximise the difference: at $x = 1$, separation $\ln 2 - 0.5 \approx 0.193147181$. (d) $m = 1/(p+1) \approx 0.284668$; $y - 1.256431209 = m(x - 2.512862417)$. (e) $\int_0^c [\ln(x+1) - 0.5x] dx = A/2$ gives $c \approx 1.128845189$. The positive bracket excludes the root at 0.

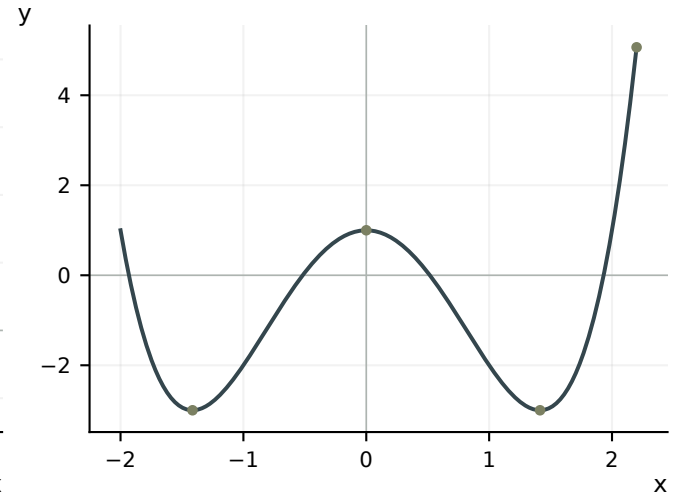
Selected graph checks: functions and areas

These are solution checks. Labels in the main answer key give the exact or numerical coordinates.

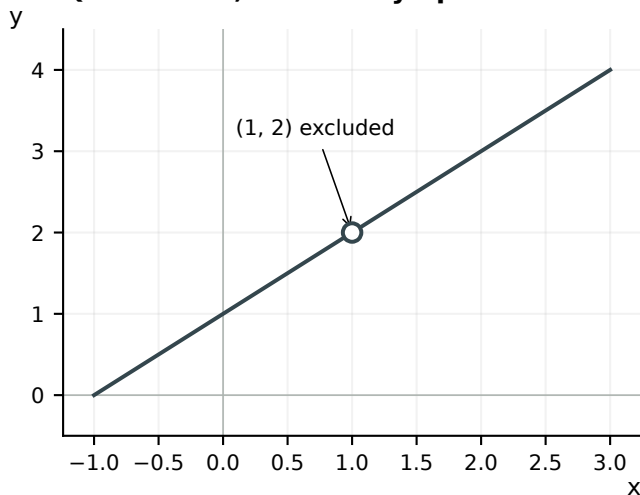
Q5 Restricted cubic



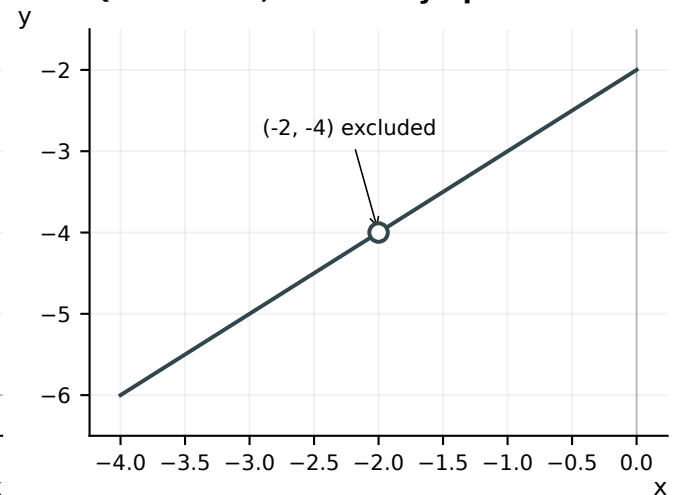
Q6 Restricted quartic



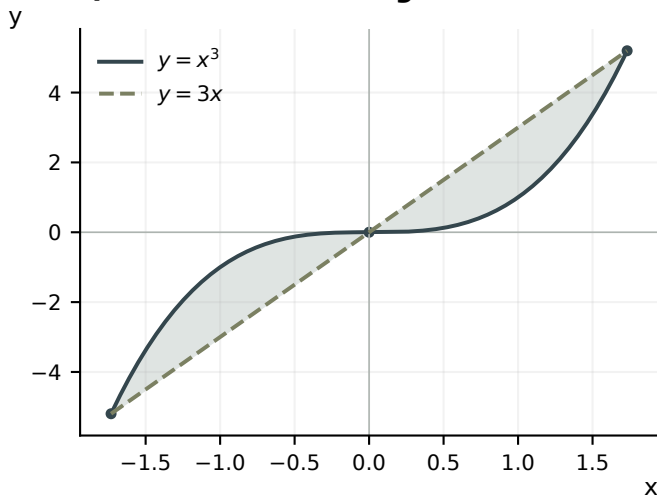
Q11 A hole, not an asymptote



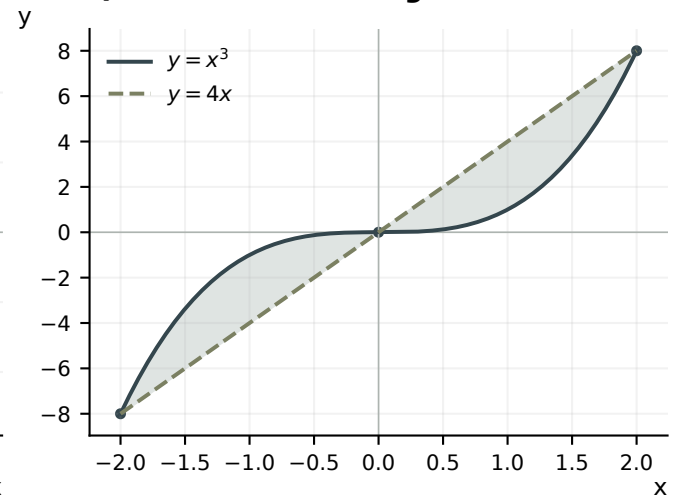
Q12 A hole, not an asymptote



Q25 Two bounded regions



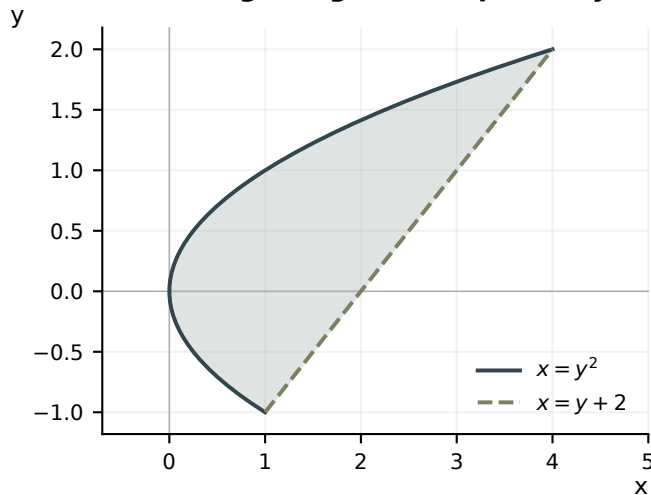
Q26 Two bounded regions



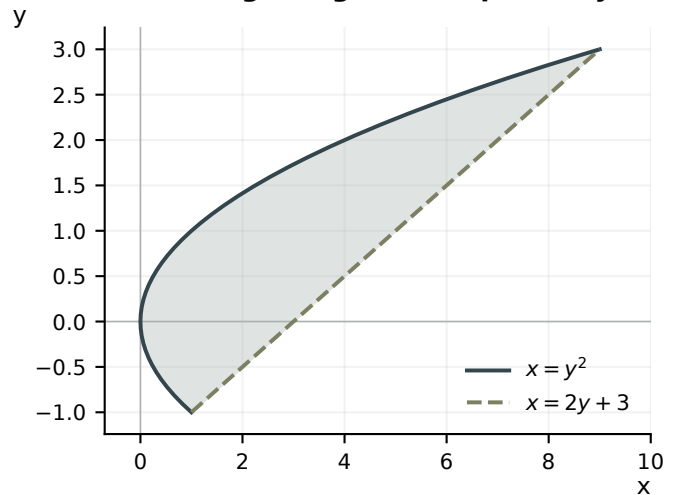
Selected graph checks: orientation and applications

For dy integrals, keep the original horizontal x -axis and vertical y -axis in the sketch.

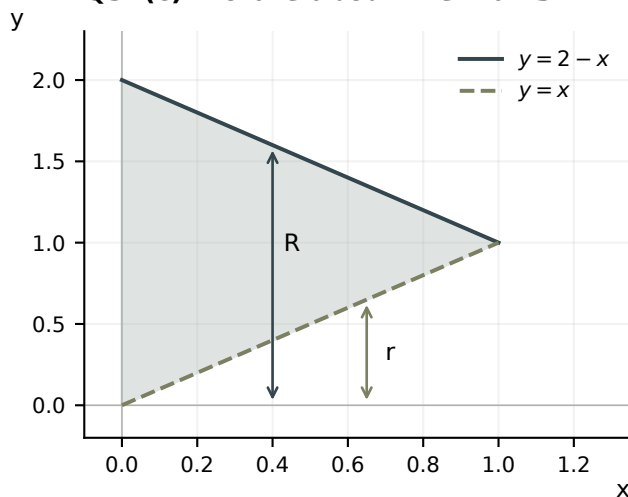
Q25 Integrating with respect to y



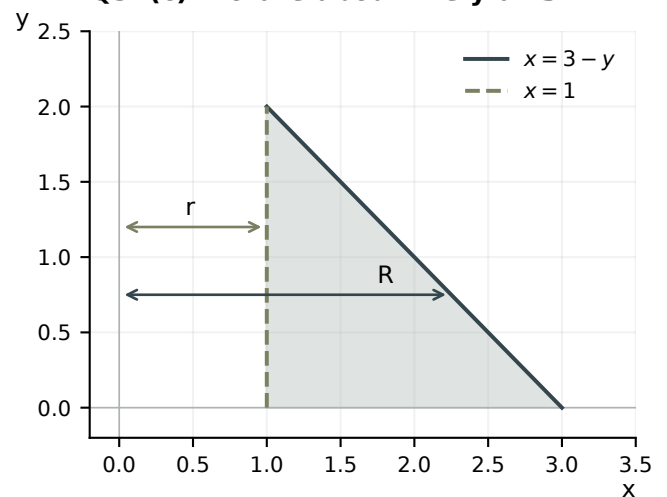
Q26 Integrating with respect to y



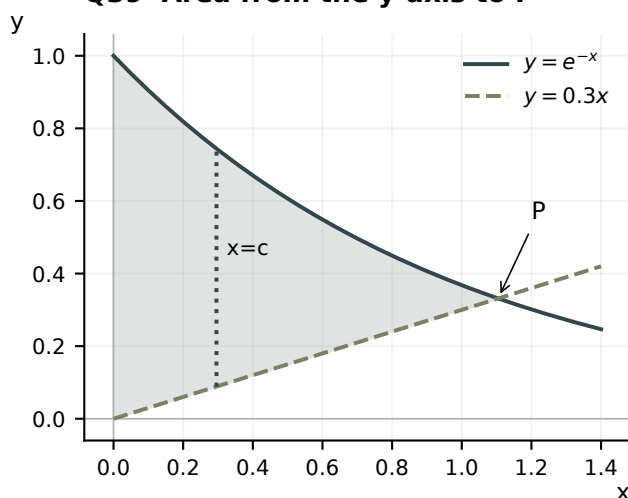
Q31(c) Rotate about the x -axis



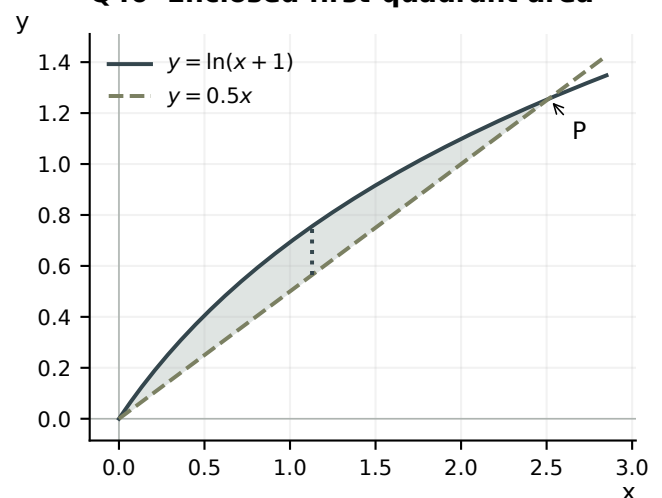
Q32(c) Rotate about the y -axis



Q39 Area from the y -axis to P



Q40 Enclosed first-quadrant area





Calculator reference and teaching notes

Working area: use Scratchpad Calculate for entries below and Scratchpad Graph for graphs and function tables. Only the spreadsheet updates in Q33–34 use a document.

Useful entry patterns

A chosen real solution	<code>nSolve($e^x=3*x^2$,$x=0.9$,0,1)</code>
All real polynomial roots	<code>polyRoots({1,0,-5,0,4})</code>
First derivative at a point	<code>nDerivative(f1(t),t=0.7)</code>
Second derivative at a point	<code>nDerivative(f1(t),t=0.7,2)</code>
Numerical integral	Use the integral template with numerical limits, or <code>nInt(f1(t),t,0,1)</code> .
Explicit Euler, step 0.2	<code>euler(x+y,x,y,{0,0.6},1,0.2,1)</code>

Before trusting an answer: check the domain, angle setting and window; search for other roots; distinguish a local extremum from a global one; check endpoint values; distinguish an area from a signed integral. A decimal is an approximation, and a failed numerical search is not proof of non-existence.

What this workbook does not claim

The non-CAS calculator does not replace exact algebraic work in Topic 2, the rules of differentiation, l'Hôpital's rule, symbolic substitution or integration by parts, symbolic differential-equation solving, or Maclaurin derivations. Questions 35–38 show numerical checks of such work. The *nDerivative* command supports first and second derivatives; higher derivatives here are first obtained by hand. No CAS-only inflexion or symbolic solve command is required.

Formula booklet use

This is a technology-practice worksheet. Any formula supplied in a question or method reminder is available for that task; its inclusion does not claim that it appears in the IB formula booklet. Students should also know how to locate the applicable formulas in their course booklet.

Source basis

Syllabus mapping was checked against the supplied *Mathematics: analysis and approaches guide*, Topic 2 (printed pp. 36–43) and Topic 5 (pp. 61–69). The original exam-style questions use the wording and subpart conventions of the supplied AA HL Paper 1 calculus compilation, adapted explicitly for calculator use. They are not official IB questions.

Texas Instruments documentation

Applying settings to Scratchpad; opening and clearing Scratchpad.

TI-Nspire CX Reference Guide (Catalog syntax for numerical derivatives, integrals, roots, piecewise functions and Euler).

Finding points of interest on a function graph; domain restrictions.

Working with function tables; solving linear systems.

Graphing differential equations and Euler settings. Documentation checked 13 October 2026.