



Financial mathematics

IB Mathematics: Analysis and Approaches SL & HL

Topic 1.4 | Compound interest, depreciation and inflation

Learning goal	Use sequences to model financial change; find values, rates and times; interpret purchasing power.
Syllabus link	AA SL 1.4 (shared with HL): compound interest, annual depreciation, real investment value with inflation, and yearly/half-yearly/quarterly/monthly compounding. Links to 1.2–1.3 follow the handout.
Structure	15 short + 15 extended + 20 IB exam-style questions; then 10 HL-exclusive challenges on the same shared content.
Technology	 GDC marks calculator questions. Q31–42 are Paper 1 style; Q43–50 are calculator / Paper 2 style. Use the TI-Nspire CX Finance Solver where requested. Logarithm methods may be completed once logarithms have been studied.

Instructions. Show your working on separate paper. Keep full precision until the final answer. Give money to the nearest cent and other approximate answers to 3 significant figures unless stated otherwise. Give exact expressions when requested. For a first year/month, give a whole number and check the adjacent values. Real values use starting-year prices. Unless stated, assume constant rates, no fees and no extra deposits or withdrawals. As in the handout, u_0 is the starting value and $S_n = u_0 + \dots + u_n$ has $n + 1$ terms. Use q for a multiplier. All exam-style questions are original.

1 | Short response

1. A savings balance is labelled $u_0, u_1, u_2, \dots$, with one term for each year. State which term represents (a) the original deposit and (b) the balance after five complete years.
2. Annual charges form the arithmetic sequence $u_0 = 120, u_1 = 135, u_2 = 150, \dots$ (dollars). Find (a) the common difference, (b) a formula for u_n and (c) u_4 .
3. An account starts at $u_0 = 2000$ and grows by 5% each year. Find (a) its multiplier q , (b) a formula for u_n and (c) u_2 .
4. Using the handout convention, write out S_3 in full and state how many terms it contains. Which symbol denotes the sum of the first six terms, starting at u_0 ?
5. Four separate annual fees are \$150, \$175, \$200 and \$225. Label the first fee u_0 . Find their total using an arithmetic-series formula, and label your answer with the correct S_n .
6. Four separate annual charges start at $u_0 = \$200$ and rise by 10% each year. Find S_3 using a geometric-series formula.
7. Five separate payments are each \$80. Label the first u_0 . State the geometric ratio and find the total using the appropriate S_n notation.



8. A single \$1000 deposit earns 10% per year, compounded annually. A student adds $u_0 + u_1 + u_2 = 1000 + 1100 + 1210$ and calls this the balance after two years. Explain the error and state (a) the correct final balance and (b) the interest earned.

9. Write the multiplier for (a) 4.5% annual compound interest and (b) 18% annual depreciation.

10. \$2000 is invested for two years at 6% per year. Find (a) the simple-interest balance, (b) the annually compounded balance and (c) the extra interest earned by compounding.

11. Complete the table for a nominal annual rate of 6% over three years. Give the rate per period as a percentage.

Compounding	k	Rate per period	Total periods
Annually	1		
Half-yearly	2		
Quarterly	4		
Monthly	12		

12. \$7500 is invested at a nominal annual rate of 3.6%, compounded monthly, for two years. Write an exact expression for the final balance. Do not evaluate it.

13. For a single investment with no regular payments, a student enters $PV = +5000$ and $FV = +6000$ in the Finance Solver. Using our point of view, correct the signs, state Pmt and explain what each sign means.

14. An investment of \$1000 grows to \$1060 in one year. Prices rise by 6% in that year. Find (a) its nominal final value and (b) its real final value in starting-year dollars.

15. The same initial deposit P can be labelled u_0 or, in another book, v_1 . Both sequences have common ratio 1.04. Write the term representing the value after n complete years in each notation.

2 | Extended response

16.  GDC \$6800 is invested at 4.2% per year, compounded annually.

(a) Find its value after seven years.

(b) Find the total interest earned.

17.  GDC \$12 500 is invested for six years at a nominal annual rate of 5.4%.

(a) Find its value if interest is compounded quarterly.

(b) Find its value if interest is compounded annually.

(c) Find the additional interest earned with quarterly compounding.



18. GDC A car depreciates at a constant percentage rate each year.

Years after purchase	0	1	2	3
Value (\$)	28 000	23 800	20 230	x

- (a) Find the annual percentage depreciation.
- (b) Find x .
- (c) Find the total percentage loss in value over the first three years.

19. GDC An investment must be worth \$12 000 in five years. It earns a nominal annual interest rate of 3.8%, compounded monthly.

- (a) Write an equation for the initial deposit P .
- (b) Find the deposit required, to the nearest cent.
- (c) Check your deposit using the Finance Solver. State all entered fields, the field solved for and the meaning of the sign of its output.

20. GDC An investment increases from \$4000 to \$5500 in eight years. The annual interest rate is constant and interest is compounded annually.

- (a) Write an equation for the annual rate $r\%$.
- (b) Find r .
- (c) Check your rate with the Finance Solver. State all entered fields and the field solved for.

21. GDC \$9000 earns 3.5% per year, compounded annually. Interest is credited only at each year-end.

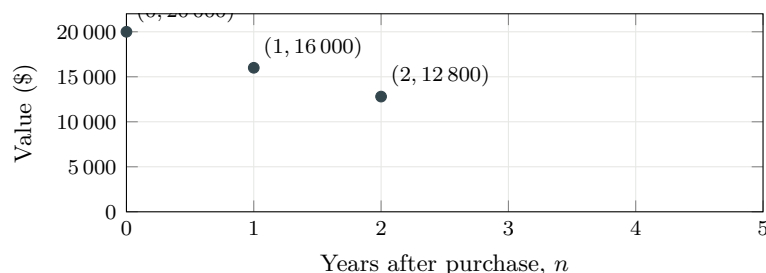
- (a) Write an inequality for the balance to reach at least \$12 500. Use logarithms (if studied) to find the boundary time.
- (b) Check the boundary time with the Finance Solver, stating the entered fields. Then give the least number of complete years required.
- (c) Check the balances immediately before and at that year-end.

22. GDC A machine costs \$24 500 and depreciates by 18% each year. Values are recorded at year-end. Find the first year-end at which its value is below \$10 000. Check the two relevant values.

23. GDC A club compares two schedules of six separate annual membership fees, payable at the starts of years 0 to 5. Schedule A starts at \$180 and adds \$12 each year. Schedule B starts at \$180 and increases by 6% each year.

- (a) Write formulas for the fees a_n and b_n .
- (b) Using the handout convention, name and calculate the total of the six fees for each schedule.
- (c) Which schedule has the smaller total, and by how much? Explain why adding these terms is appropriate.

24.  GDC The graph shows the recorded value of a machine at the purchase time and the first two year-ends. Assume constant annual percentage depreciation.



- (a) Find the annual percentage depreciation and write a formula for u_n .
- (b) Calculate u_4 and add the point to the graph.
- (c) Find the first year-end at which the machine is worth less than half its purchase price.
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25.  GDC \$18 000 earns 4% per year, compounded annually, for seven years. Annual inflation is 2.5%.
- (a) Find the final nominal value (the actual account balance).
- (b) Find its real value in starting-year dollars.
- (c) Find the total percentage increase in purchasing power.
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26.  GDC \$10 000 earns 3.2% per year, compounded annually, for five years. Annual inflation is 4.5%.
- (a) Find the nominal and real final values, using starting-year dollars for the real value.
- (b) Explain how the balance can increase while purchasing power decreases.
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27.  GDC \$6200 earns 3% per year for two years, then 4.2% per year for three more years. Interest is compounded annually throughout.
- (a) Find the balance after all five years.
- (b) Explain why $6200(1.042)^5$ is not an appropriate model.
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28.  GDC A camera costs \$1600 today and its price increases by 3% each year. A student invests \$1300 today at 4.5% per year, compounded annually, with no further deposits.
- (a) Can the student afford the camera after five years? Justify your answer.
- (b) Find the first year-end at which the savings are sufficient.
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29.  GDC An account earns 7% per year, compounded annually, while prices rise by 3% per year. A student claims that the real annual return is exactly 4%.
- (a) Calculate the exact real annual growth factor and the real annual percentage return.
- (b) Explain why subtracting the two rates gives only an approximation.



30. GDC \$8000 earns a nominal annual interest rate of 4.8%, compounded monthly. A finance solver uses one period per month.

- (a) For a three-year investment, state N , $I(\%)$, PV , Pmt , PpY , CpY and $PmtAt$, using a negative PV . Then find FV .
- (b) Use logarithms (if studied), then check by solving for N , to find the least number of complete months needed to reach \$10 000.

3 | IB exam-style questions

Questions 31–42: Paper 1 style (no calculator). Marks are shown in square brackets.

31. [Maximum mark: 4] Maya invests \$1000 at 10% per year, compounded annually.

- (a) Find the balance after two years. [2]
- (b) Find the total interest earned over the two years. [1]
- (c) Write an expression for the balance after n years. [1]

32. [Maximum mark: 5] A computer costs \$5000. It depreciates by 20% each year.

- (a) Find its value after two years. [2]
- (b) Find the total percentage decrease over those two years. [2]
- (c) Explain why the total percentage decrease is not 40%. [1]

33. [Maximum mark: 4] A deposit of P dollars earns 20% per year, compounded annually. After two years it is worth \$4320.

- (a) Find P . [3]
- (b) Find the interest earned in the first year. [1]

34. [Maximum mark: 4] A machine retains 90% of its value each year. Its value at the end of year 2 is \$8100.

- (a) Find its value when new. [2]
- (b) Find its value at the end of year 3. [1]
- (c) Find the total loss in value from purchase to the end of year 3. [1]

35. [Maximum mark: 5] Two accounts each start with P dollars. Both quote an annual rate r , written as a decimal. Account A uses simple interest; account B compounds annually.

- (a) Write their balances after two years. [2]
- (b) Show that B exceeds A by Pr^2 dollars after two years. [2]
- (c) Hence find this difference when $P = 10\,000$ and $r = 0.10$. [1]

36. [Maximum mark: 5] A bank advertises a nominal annual interest rate of 8%, compounded quarterly. Leo invests \$2500.

- (a) State the quarterly growth factor. [1]
- (b) Write an exact expression for the balance after two years. [2]
- (c) Write an exact expression for the effective annual percentage rate. [2]



37. [Maximum mark: 5] Two accounts each start with \$1000. A pays 20% per year compounded annually. B pays a nominal annual rate of 20% compounded half-yearly.

- (a) Find both balances after one year. [3]
- (b) Find the extra interest earned by B in that year. [1]
- (c) Explain why the balances differ although both quoted rates are 20%. [1]

38. [Maximum mark: 5] An investment of \$2000 earns 21% over one year. Prices rise by 10% during that year.

- (a) Find the final nominal value. [1]
- (b) Find the real value in starting-year dollars. [2]
- (c) Find the percentage increase in purchasing power. [2]

39. [Maximum mark: 4] An investment starts at P dollars. Its annual compound interest rate and the annual inflation rate are both r , written as a decimal, with $r > 0$.

- (a) Write its nominal value after n years. [1]
- (b) Show that its real value in starting-year dollars is unchanged. [2]
- (c) Explain what this means about purchasing power. [1]

40. [Maximum mark: 5] A fund worth \$10 000 increases by 10% in year 1, then decreases by 10% in year 2.

- (a) Find its value after year 2. [2]
- (b) Find its overall percentage change. [2]
- (c) Explain why the two changes do not cancel. [1]

41. [Maximum mark: 5] An investment has an annual compound growth factor of $2^{1/5}$. It starts at P dollars.

- (a) Show that its value doubles after five years. [2]
- (b) Find the first year-end when its value is at least $4P$. [2]
- (c) Write the annual percentage interest rate exactly. [1]

42. [Maximum mark: 5] A one-off fee of 5% is deducted immediately from a deposit of \$2000. The remaining amount earns 10% per year, compounded annually, for two years.

- (a) Find the amount invested after the fee. [1]
- (b) Find the balance after two years. [2]
- (c) Find how much lower this is than the balance with no fee. [2]



3 | IB exam-style questions: calculator practice

Questions 43–50: calculator / Paper 2 style.

43. [Maximum mark: 6] A university course costs \$25 000 today. Its cost increases by 3.4% each year. A family saves for the fee due in eight years, using an account paying a nominal annual rate of 4.1%, compounded quarterly.

- (a) Find the fee due in eight years. [2]
- (b) Find the single deposit required today to meet that fee. [3]
- (c) State one limitation of this model. [1]

44. [Maximum mark: 5] A vehicle costs \$36 500. It loses 14% of its value in its first year, then 9% of its current value in each subsequent year.

- (a) Find its value at the end of year 1. [1]
- (b) Find its value at the end of year 5. [2]
- (c) Find the total percentage depreciation over the first five years. [2]

45. [Maximum mark: 6] A saver compares the following accounts. There are no fees.

Account	Nominal annual rate	Compounding
A	5.1%	Quarterly
B	5.0%	Monthly

- (a) Find the effective annual percentage rate of each account. [3]
- (b) \$18 000 is invested for ten years. Determine which account gives the larger balance and find the difference. [3]

46. [Maximum mark: 6] A house costs \$275 000. Its value increases by 2.8% per year for 12 years, while annual inflation is 3.1%.

- (a) Find its nominal value after 12 years. [2]
- (b) Find its real value in purchase-year dollars. [2]
- (c) Find the total percentage change in purchasing power and interpret your answer. [2]

47. [Maximum mark: 7] \$12 000 grows to \$15 900 in six years at a constant nominal annual interest rate of $r\%$, compounded quarterly.

- (a) Write an equation for r . [2]
- (b) Find r . [3]
- (c) Find the effective annual percentage rate. [2]

48. [Maximum mark: 7] Two machines are purchased at the same time. A costs \$30 000 and depreciates by 12% annually. B costs \$22 000 and depreciates by 6% annually.

- (a) Write expressions for their values after n complete years. [2]
- (b) Find the first year-end at which B is worth more than A. [3]
- (c) Check your answer using the values at that year-end and the preceding year-end. [2]



49.  GDC [Maximum mark: 8] An investment earns a constant annual compound interest rate.

Years after investment	0	4	8
Balance (\$)	8000	9724.05	x

- (a) Find the annual percentage interest rate. [3]
- (b) Find x . [2]
- (c) Find the least number of complete years needed for the balance to reach \$15 000. [3]

50.  GDC [Maximum mark: 7] A loan of \$7000 is charged a nominal annual interest rate of 7.2%, compounded monthly. No payments are made for 18 months. Immediately after the interest for month 18 is added, \$2500 is repaid. No further payments are made for six months.

- (a) Find the debt just before the repayment. [2]
- (b) Find the debt just after the repayment. [1]
- (c) Find the debt at the end of month 24. [2]
- (d) Find the total interest charged over the 24 months. [2]

**4 | AA HL exclusive challenge**

Shared SL/HL questions end at Q50. This section is intended for HL students: harder applications of the same 1.4 content, with no additional AHL syllabus requirement.

51. GDC A fund increases by $r\%$ in one year, then decreases by $r\%$ in the next, where $0 < r < 100$. Its value after the two years is 91% of its initial value.

- (a) Find r .
- (b) Find the percentage increase required in year 3 to restore the original value.

52. GDC \$3000 is invested at time $t = 0$. A further X dollars is deposited exactly two years later, just after interest is added. The account earns 4% per year, compounded annually, and is worth \$10 000 at the end of year 7.

- (a) Write an equation for X , showing how long each deposit earns interest.
- (b) Find X .

53. GDC An account earns a fixed annual compound interest rate. Two records are available.

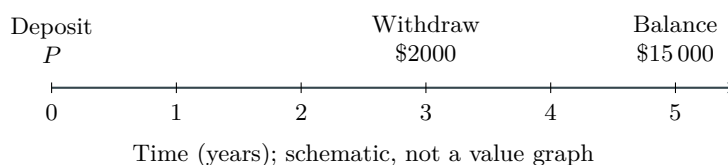
Years after opening	2	5
Balance (\$)	12 100	16 105.10

- (a) Find the annual growth factor and the original deposit.
- (b) Find the first year-end at which the balance exceeds \$26 000.

54. GDC An investor wants \$15 000 today to have purchasing power of at least \$20 000 in today's dollars after ten years. Inflation is 2% per year. Interest is compounded monthly at a fixed nominal annual rate of $r\%$.

- (a) Form an inequality for r .
- (b) Find the threshold value of r to four decimal places.
- (c) If the bank quotes rates in steps of 0.01 percentage points, find the minimum suitable quoted rate.

55. GDC An initial deposit P earns 5% per year, compounded annually. \$2000 is withdrawn immediately after the interest at the end of year 3. At the end of year 5 the balance is \$15 000.



- (a) Write one equation connecting P with the final balance.
- (b) Find P .
- (c) Explain the exponent attached to the withdrawn \$2000 if you expand your equation.



56. Two accounts each start with \$3000. A earns 20% in year 1 and $100r\%$ in year 2. B earns $100r\%$ in year 1 and 20% in year 2, where $r \geq 0$ is a decimal rate.

- (a) Show that their two-year balances would be equal if there were no fees.
- (b) A fixed fee of \$200 is instead deducted from each account just after the first year's interest. Show that the final balance of A minus that of B is $40 - 200r$ dollars.
- (c) Hence find the values of r for which A finishes with more money.

57. _{GDC} \$8000 earns 5% per year, compounded annually. Inflation is 2% per year.

- (a) Find the first year-end when its real value, in starting-year dollars, is at least \$10 000.
- (b) Find the first year-end when its nominal value is at least \$10 000, and explain the difference.

58. _{GDC} \$20 000 is invested at a nominal annual interest rate of 6%, compounded quarterly. After three years its real value is \$21 200 in starting-year dollars. Annual inflation has been constant at $h\%$.

- (a) Find h .
- (b) Find the effective annual percentage growth in purchasing power.

59. _{GDC} A machine costs \$45 000 and loses 12% of its current value each year. A new replacement costs \$45 000 today, but its price rises by 3% each year. The old machine can be sold for its recorded year-end value.

- (a) Write an expression for the additional cash needed to replace the machine at the end of year n .
- (b) Find the first year-end when the additional cash needed exceeds \$30 000.
- (c) Explain why there can be only one first crossing of this threshold.

60. _{GDC} Account A earns 6% interest each year; a fee of 1% of the new balance is then deducted at each year-end. Account B earns 4.9% per year with no fees. Both start with \$10 000.

- (a) Find A's effective annual growth factor after fees.
- (b) Compare their balances after ten years.
- (c) If A's gross annual interest rate were instead $r\%$, find the exact threshold above which A would outperform B.
- (d) With A's original 6% gross rate and annual inflation of 2%, find its real value after ten years in starting-year dollars.

**Answer key and solution outlines**

Use unrounded values between steps. Here, nominal value means the actual dollar balance; a nominal annual interest rate is the quoted rate before allowing for compounding frequency.

1 | Short response

1. (a) u_0 . (b) u_5 : five years means five steps from index 0.
2. (a) $d = 15$. (b) $u_n = u_0 + nd = 120 + 15n$. (c) $u_4 = 180$ dollars.
3. (a) $q = 1.05$. (b) $u_n = u_0 q^n = 2000(1.05)^n$. (c) $u_2 = \$2205$.
4. $S_3 = u_0 + u_1 + u_2 + u_3$: four terms. The first six terms end at u_5 , so their sum is S_5 .
5. There are $3 + 1 = 4$ terms. $S_3 = \frac{3+1}{2}(u_0 + u_3) = \frac{4}{2}(150 + 225) = \$750$.
6. $S_3 = u_0(q^4 - 1)/(q - 1) = 200(1.1^4 - 1)/0.1 = \$928.20$. This adds $200 + 220 + 242 + 266.20$.
7. $q = 1$, so use $S_n = (n + 1)u_0$. Five terms give $S_4 = 5(80) = \$400$.
8. The balances count the same invested money repeatedly; they are not separate payments. (a) Final balance $= u_2 = \$1210$. (b) Interest $= u_2 - u_0 = \$210$.
9. (a) $1 + 4.5/100 = 1.045$. (b) $1 - 18/100 = 0.82$.
10. (a) $u_0(1 + nr/100) = 2000(1 + 2(0.06)) = \$2240$. (b) $u_0 q^2 = 2000(1.06)^2 \approx \$2247.20$. (c) $\$7.20$.
11. Annually: 6%, 3 periods. Half-yearly: 3%, 6 periods. Quarterly: 1.5%, 12 periods. Monthly: 0.5%, 36 periods.
12. $FV = PV(1 + r/(100k))^{kn} = 7500(1 + 3.6/1200)^{24} = 7500(1.003)^{24}$.
13. $PV = -5000$: our deposit leaves us. $FV = +6000$: the money returns to us. $Pmt = 0$. The minus sign describes cash-flow direction, not a loss in the account.
14. (a) $\$1060$. (b) Real value $= 1060/1.06 = \$1000$; purchasing power is unchanged.
15. $u_n = P(1.04)^n$ and $v_{n+1} = P(1.04)^n$. Both involve n multiplications, irrespective of the starting label.

2 | Extended response

16. (a) $V = 6800(1.042)^7 \approx \$9\,069.49$. (b) $V - 6800 \approx \$2\,269.49$.
17. (a) $12\,500(1 + 0.054/4)^{24} \approx \$17\,245.67$. (b) $12\,500(1.054)^6 \approx \$17\,137.75$. (c) Difference $\approx \$107.93$.
18. (a) Factor $= 23\,800/28\,000 = 0.85$, so 15%. (b) $x = 28\,000(0.85)^3 \approx \$17\,195.50$. (c) $100(1 - 0.85^3) = 38.5875\% \approx 38.6\%$.
19. (a) Use $FV = PV(1 + r/(100k))^{kn}$ with positive balances: $12\,000 = P(1 + 3.8/1200)^{60}$. (b) $P = 12\,000/(1 + 3.8/1200)^{60} \approx \$9926.49$. (c) $N = 60$, $I(\%) = 3.8$, $Pmt = 0$, $FV = +12\,000$, $PpY = CpY = 12$, $PmtAt = \text{END}$; solve $PV \approx -9926.49$. The deposit leaves us; its amount is positive.
20. (a) $5500 = 4000(1 + r/100)^8$. (b) $(5500/4000)^{1/8} = 1 + r/100$, so $r = 100[(5500/4000)^{1/8} - 1] \approx 4.06$: the rate is 4.06%. (c) $N = 8$, $PV = -4000$, $Pmt = 0$, $FV = +5500$, $PpY = CpY = 1$, $PmtAt = \text{END}$; solve $I(\%)$.
21. (a) $9000(1.035)^n \geq 12\,500$, so $n \geq \ln(12\,500/9000)/\ln(1.035) = 9.549\dots$. (b) $I(\%) = 3.5$, $PV = -9000$, $Pmt = 0$, $FV = 12\,500$, $PpY = CpY = 1$, $PmtAt = \text{END}$; solve $N = 9.549\dots$. Thus 10 complete years. (c) $u_9 \approx \$12\,266.08 < \$12\,500$ and $u_{10} \approx \$12\,695.39 > \$12\,500$.
22. $24\,500(0.82)^n < 10\,000$. Since $\ln(0.82) < 0$, $n > \ln(10\,000/24\,500)/\ln(0.82) = 4.515\dots$. First: year 5. Values: $u_4 \approx \$11\,076.98$, $u_5 \approx \$9\,083.13$.
23. (a) $a_n = 180 + 12n$; $b_n = 180(1.06)^n$. (b) Both totals end at index 5: $S_5^{(A)} = \frac{6}{2}[2(180) + 5(12)] = \$1260$; $S_5^{(B)} = 180(1.06^6 - 1)/0.06 \approx \$1255.56$. (c) B costs $\$4.44$ less. These are six distinct fees, so summing them gives the total paid.

**2 | Extended response (continued)**

- 24.** (a) Factor = $16\,000/20\,000 = 0.8$, so 20% depreciation; $u_n = 20\,000(0.8)^n$. (b) $u_4 = \$8192$; plot (4, 8192). (c) $u_3 = \$10\,240 > \$10\,000$ and $V_4 < \$10\,000$: year 4.
- 25.** (a) $18\,000(1.04)^7 \approx \$23\,686.77$. (b) Divide by the price factor: $18\,000(1.04/1.025)^7 \approx \$19\,926.86$. (c) $100[(1.04/1.025)^7 - 1] \approx 10.7\%$.
- 26.** (a) Nominal = $10\,000(1.032)^5 \approx \$11\,705.73$; real = $10\,000(1.032/1.045)^5 \approx \$9\,393.27$. (b) Prices grow faster than the balance, so the final money buys less than the original \$10 000.
- 27.** (a) $6200(1.03)^2(1.042)^3 \approx \$7\,441.65$. (b) The first two years earn 3%, not 4.2%. Each period needs its own growth factor.
- 28.** (a) Savings = $1300(1.045)^5 \approx \$1\,620.04$; price = $1600(1.03)^5 \approx \$1\,854.84$: no. (b) $(1.045/1.03)^n \geq 1600/1300$ gives $n \geq 14.361\dots$, hence 15 years. At year 14 the savings/price ratio is 0.995; at year 15 it is 1.01.
- 29.** (a) Factor = $1.07/1.03 = 107/103$; real return = $100(107/103 - 1) = 400/103\% = 3.883495\dots\% \approx 3.88\%$. (b) Purchasing power requires division by the price factor. The difference from 4% is about 0.117 percentage points.
- 30.** (a) $N = 36$, $I(\%) = 4.8$, $PV = -8000$, $Pmt = 0$, $PpY = CpY = 12$, $PmtAt = END$. Solve $FV = 8000(1.004)^{36} \approx \$9236.42$. (b) With m in months, $8000(1.004)^m \geq 10\,000$, giving $m \geq \ln(1.25)/\ln(1.004) = 55.897\dots$. In the solver change FV to 10 000 and solve $N = 55.897\dots$. Hence 56 months (4 years 8 months); $u_{55} \approx \$9964.24$ and $u_{56} \approx \$10\,004.10$.

3 | IB exam-style questions

- 31.** (a) $1000(1.1)^2 = \$1210$. (b) $1210 - 1000 = \$210$. (c) $1000(1.1)^n$.
- 32.** (a) $5000(0.8)^2 = \$3200$. (b) $100(5000 - 3200)/5000 = 36\%$. (c) The second 20% decrease is applied to \$4000, a smaller base.
- 33.** (a) $4320 = P(1.2)^2$, so $P = 4320/1.44 = \$3000$. (b) $0.2(3000) = \$600$.
- 34.** (a) $P(0.9)^2 = 8100$, hence $P = \$10\,000$. (b) $8100(0.9) = \$7290$. (c) $10\,000 - 7290 = \$2710$.
- 35.** (a) $A = P(1 + 2r)$, $B = P(1 + r)^2$. (b) $B - A = P(1 + 2r + r^2 - 1 - 2r) = Pr^2$. (c) $10\,000(0.10)^2 = \$100$.
- 36.** (a) $1 + 0.08/4 = 1.02$. (b) $2500(1.02)^8$. (c) $100[(1.02)^4 - 1]\%$. This is the actual percentage growth over one year.
- 37.** (a) $A = 1000(1.2) = \$1200$; $B = 1000(1.1)^2 = \$1210$. (b) \$10. (c) B earns interest on the first half-year's interest during the second half-year.
- 38.** (a) $2000(1.21) = \$2420$. (b) $2420/1.1 = \$2200$. (c) $100(2200 - 2000)/2000 = 10\%$.
- 39.** (a) $P(1 + r)^n$. (b) Real value = $P(1 + r)^n/(1 + r)^n = P$. (c) It can buy the same quantity of goods as at the start, under this inflation model.
- 40.** (a) $10\,000(1.1)(0.9) = \$9900$. (b) $100(9900 - 10\,000)/10\,000 = -1\%$: a 1% decrease. (c) The decrease acts on \$11 000, whereas the increase acted on \$10 000.
- 41.** (a) $P(2^{1/5})^5 = 2P$. (b) $P2^{n/5} \geq P2^2$ gives $n \geq 10$; first year-end is 10. (c) $100(2^{1/5} - 1)\%$.
- 42.** (a) $2000(0.95) = \$1900$. (b) $1900(1.1)^2 = \$2299$. (c) No-fee balance = $2000(1.1)^2 = \$2420$; difference = \$121. The fee also loses the interest it could have earned.
- 43.** (a) $25\,000(1.034)^8 \approx \$32\,666.63$. (b) $P(1 + 0.041/4)^{32} = 25\,000(1.034)^8$, so $P \approx \$23\,571.15$. (c) For example, the future fee increases or interest rates may differ from the assumed constant rates.
- 44.** (a) $36\,500(0.86) \approx \$31\,390.00$. (b) $36\,500(0.86)(0.91)^4 \approx \$21\,525.68$. (c) $100[1 - 0.86(0.91)^4] \approx 41.0\%$.
- 45.** (a) $i_A = 100[(1 + 0.051/4)^4 - 1] \approx 5.20\%$; $i_B = 100[(1 + 0.05/12)^{12} - 1] = 5.12\%$. (b) $A = 18\,000(1 + 0.051/4)^{40} \approx \$29\,878.76$; $B = 18\,000(1 + 0.05/12)^{120} \approx \$29\,646.17$. A exceeds B by \$232.59.
- 46.** (a) $275\,000(1.028)^{12} \approx \$383\,045.24$. (b) $275\,000(1.028/1.031)^{12} \approx \$265\,549.87$. (c) $100[(1.028/1.031)^{12} - 1] = -3.44\%$. Purchasing power falls by about 3.44%, despite the higher nominal value.

**3 | IB exam-style questions (continued)**

- 47.** (a) $15\,900 = 12\,000(1 + r/400)^{24}$. (b) $r = 400[(15\,900/12\,000)^{1/24} - 1] = 4.72\%$. (c) $100[(15\,900/12\,000)^{1/6} - 1] \approx 4.80\%$.
- 48.** (a) $A_n = 30\,000(0.88)^n$, $B_n = 22\,000(0.94)^n$. (b) $(0.94/0.88)^n > 30\,000/22\,000$ gives $n > 4.7$, so year 5. (c) Year 4: $A \approx \$17\,990.86$, $B \approx \$17\,176.48$; year 5: $A \approx \$15\,831.96$, $B \approx \$16\,145.89$.
- 49.** (a) $8000(1 + r)^4 = 9724.05$, so $r = (9724.05/8000)^{1/4} - 1 = 0.05$: 5%. (b) $x = 8000(1.05)^8 \approx \$11\,819.64$. (c) $n \geq \ln(15\,000/8000)/\ln(1.05) = 12.9$: 13 years. $V_{12} \approx \$14\,366.85$, $V_{13} \approx \$15\,085.19$.
- 50.** (a) $D_{18} = 7000(1.006)^{18} \approx \$7795.82$. (b) $D_{18} - 2500 \approx \$5\,295.82$. (c) $D_{24} = [7000(1.006)^{18} - 2500](1.006)^6 \approx \$5\,489.35$. (d) Interest = $D_{24} + 2500 - 7000 \approx \$989.35$.

4 | AA HL exclusive challenge

- 51.** (a) $(1 + r/100)(1 - r/100) = 0.91$, hence $(r/100)^2 = 0.09$ and $r = 30$. (b) Recovery factor = $1/0.91$; increase = $100(1/0.91 - 1) = 9.8901\dots\% \approx 9.89\%$.
- 52.** (a) The first deposit earns for 7 years and the second for 5: $3000(1.04)^7 + X(1.04)^5 = 10\,000$. (b) $X = [10\,000 - 3000(1.04)^7]/(1.04)^5 \approx \$4974.47$.
- 53.** (a) $Pq^2 = 12\,100$, $Pq^5 = 16\,105.10$. Dividing gives $q^3 = 1.331$, so $q = 1.1$ and $P = 12\,100/1.1^2 = \$10\,000$. (b) $1.1^n > 2.6$ gives $n > \ln(2.6)/\ln(1.1) = 10.025\dots$: year 11. $V_{10} \approx \$25\,937.42$, $V_{11} \approx \$28\,531.17$.
- 54.** (a) $15\,000(1 + r/1200)^{120}/(1.02)^{10} \geq 20\,000$. (b) $r \geq 1200[((20\,000/15\,000)(1.02)^{10})^{1/120} - 1] \approx 4.8669$. (c) Round up to the next available rate: 4.87%.
- 55.** (a) $[P(1.05)^3 - 2000](1.05)^2 = 15\,000$. (b) $P = [15\,000/(1.05)^2 + 2000]/(1.05)^3 \approx \$13\,480.57$. (c) Expanded: $P(1.05)^5 - 2000(1.05)^2 = 15\,000$. The withdrawal removes money that would otherwise earn interest for the final two years.
- 56.** (a) Both give $3000(1.2)(1 + r)$ because multiplication commutes. (b) $A = (3600 - 200)(1 + r)$; $B = [3000(1 + r) - 200](1.2)$. Thus $A - B = 3400 + 3400r - (3360 + 3600r) = 40 - 200r$. (c) $40 - 200r > 0$ gives $0 \leq r < 0.2$, i.e. a rate below 20%.
- 57.** (a) $8000(1.05/1.02)^n \geq 10\,000$ gives $n \geq 7.7$: 8 years. Real values at years 7 and 8: $\$9\,799.72$ and $\$10\,087.95$. (b) $n \geq \ln(1.25)/\ln(1.05) = 4.57$: 5 years. Nominal values at years 4 and 5: $\$9\,724.05$ and $\$10\,210.25$. Rising prices delay the purchasing-power target.
- 58.** (a) $20\,000(1.015)^{12}/(1 + h/100)^3 = 21\,200$, so $h = 100[((1.015)^{12}/1.06)^{1/3} - 1] = 4.09\%$. (b) Annual real factor = $(21\,200/20\,000)^{1/3}$; return = $100(1.06^{1/3} - 1) = 1.96\%$.
- 59.** (a) $C_n = 45\,000[(1.03)^n - (0.88)^n]$. (b) A table or graph gives $C_5 \approx \$28\,419.40$ and $C_6 \approx \$32\,834.17$: year 6. (c) The replacement price strictly increases and the resale value strictly decreases, so C_n strictly increases.
- 60.** (a) $q_A = 1.06(0.99) = 1.0494$ (a 4.94% net annual return). (b) $A = 10\,000(1.0494)^{10} \approx \$16\,196.11$; $B = 10\,000(1.049)^{10} \approx \$16\,134.48$. A is higher by $\$61.63$. (c) $0.99(1 + r/100) > 1.049$, so $r > 100(1.049/0.99 - 1) = 590/99 \approx 5.9596$. Equality gives equal balances. (d) $10\,000(1.0494/1.02)^{10} \approx \$13\,286.45$.