

Rational exponents and logarithms

AA SL & HL Mathematics | Worksheet 1.7

60 questions: fluency, applications, Paper 1 practice and HL challenges

Overview

Learning goal: simplify rational powers, apply logarithm laws and change of base, and solve exponential equations.

Syllabus link: AA SL 1.7, shared by SL and HL. The HL challenge section deepens this shared content; no additional AHL syllabus code is introduced.

Structure: Q1–15 short response; Q16–30 extended response; Q31–50 original Paper 1-style questions; Q51–60 HL-exclusive challenges.

Calculator guidance: only Q24–26 use a GDC. All other questions, including Paper 1 and HL challenges, are non-calculator.

Instructions

Work on separate paper and show your reasoning. Give exact answers unless an approximation is requested. Check logarithm arguments and bases before accepting solutions.

Formula booklet: logarithm laws and change of base are provided; rational-exponent laws are not. For GDC work, use TI-Nspire CX (non-CAS) **Scratchpad Calculate** and round only at the end.

1 Short response

Non-calculator

1. Evaluate exactly.

(a) $81^{3/4}$

(b) $64^{-2/3}$

(c) $\left(\frac{16}{81}\right)^{-3/4}$

2. Let $a > 0$. Write each expression as a single power of a .

(a) $\sqrt[6]{a^5}$

(b) $\frac{1}{\sqrt[3]{a^2}}$

3. For $x > 0$, simplify.

(a) $\frac{x^{5/6}x^{1/3}}{x^{1/2}}$

(b) $(8x^6)^{2/3}$

4. Evaluate exactly.

(a) $\log_7 49$

(b) $\log_5 \left(\frac{1}{125} \right)$

(c) $\ln \sqrt{e}$



5. Use the laws of logarithms to evaluate.

(a) $\log_3 12 + \log_3 \left(\frac{9}{4}\right)$

(b) $\log_5 250 - \log_5 2$

6. For $x, y > 0$, expand $\ln \left(\frac{5x^3}{\sqrt{y}}\right)$.

7. For $x, y > 0$, write $3\log_2 x - \log_2 y + 2$ as a single logarithm.

8. Evaluate exactly by expressing both the base and the argument as powers of the same number.

(a) $\log_9 27$

(b) $\log_{1/4} 8$

9. Use change of base.

(a) Write $\log_7 12$ using natural logarithms.

(b) Write $\frac{1}{\log_2 5}$ as a single logarithm.

10. Solve $4^{x+1} = 32$.

11. Solve $9^{2x-1} = 27^{x+1}$.

12. Solve $5^x = 14$, giving an exact answer.

13. Solve $e^{3x-2} = 7$, giving an exact answer.

14. State the set of real values of x for which each expression is defined.

(a) $\ln(2x - 5)$

(b) $\log_4(7 - x)$



15. A student writes $\log_{10}(2 + 8) = \log_{10} 2 + \log_{10} 8$. Explain the error and simplify each side correctly.

2 Extended response

Non-calculator except Q24–26, labelled [GDC].

16. The table shows values of $y = 3^x$.

x	-2	$-\frac{1}{2}$	0	$\frac{3}{2}$
y	A	B	1	C

(a) Find the missing exact values A , B and C .

(b) For the same rule, find x when $y = \frac{1}{\sqrt{27}}$.

17. Let $x > 0$ and $E = \frac{\sqrt[3]{x^2}\sqrt{x}}{x^{1/6}}$.

(a) Show that $E = x$.

(b) Hence solve $\frac{\sqrt[3]{x^2}\sqrt{x}}{x^{1/6}} = \sqrt[4]{81}$.

18. Let $p = \log_2 3$ and $q = \log_2 5$. Express each answer in terms of p and q .

(a) $\log_2 45$

(b) $\log_6 25$

19. Consider $\log_3(x - 1) + \log_3(x + 1) = 2$.

(a) State the domain restriction and combine the logarithms.

(b) Solve the equation, explaining any rejected value.

20. Consider $\ln(x + 6) - \ln x = \ln 4$.

(a) State the domain restriction.

(b) Solve the equation.



21. Solve $2 \log_5 x = \log_5(6x - 5)$. State the domain and check all candidates.

22. Consider $3^{2x} - 10 \cdot 3^x + 9 = 0$.

(a) Use $u = 3^x$ to obtain a quadratic equation.

(b) Hence find all real values of x .

23. Solve $2^{x+2} = 5^{x-1}$.

(a) Take natural logarithms and collect the terms containing x .

(b) Give the exact solution as a ratio of two logarithms.

24. [GDC] Use a calculator to evaluate each expression to three significant figures. Show the change-of-base expression entered.

(a) $\log_6 17$

(b) $\log_{0.4} 7$

25. [GDC] A culture has population $P(t) = 600(1.18)^t$, where t is measured in hours.

(a) Find the population after 5 hours, to the nearest whole organism.

(b) Find the time when the population first reaches 1500. Give an exact expression and an answer to three significant figures.

26. [GDC] The mass of a substance is $M(t) = 80e^{-0.12t}$ grams, where t is in days.

(a) Find the time taken for the mass to halve.

(b) Find the time taken for the mass to fall to 10 grams. Relate this to your answer to part (a). Give times to three significant figures.

27. An exponential model has the form $y = Ab^t$, with $A > 0$ and $b > 0$.

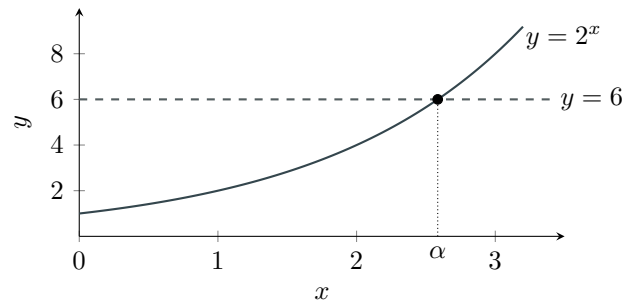
t	0	2
y	5	45

- Use the table to find A and b .
- Find the exact value of t for which $y = 20$.

28. Consider $\log_2 x + \log_4 x = 6$, where $x > 0$.

- Write $\log_4 x$ in terms of $\log_2 x$.
- Hence solve the equation.

29. The diagram shows $y = 2^x$ and $y = 6$. Their intersection has x -coordinate α .



- Use exact powers of 2 to show that $2 < \alpha < 3$.
- Find α exactly.

30. A student solves $\ln(x - 2) + \ln(x + 2) = \ln 5$ and gives $x = \pm 3$.

- Identify the missing check.
- Give the complete correct solution.

**3 IB exam-style / Paper 1**

Non-calculator. Original questions; marks are suggested practice allocations.

31. [Maximum mark: 4]

Let $a = 64^{1/3}$ and $b = 16^{-1/2}$.

- (a) Find the values of a and b . [2 marks]
- (b) Hence evaluate $(ab)^{3/2}$. [2 marks]

32. [Maximum mark: 6]

Let $x > 0$ and $F = \frac{(27x^6)^{2/3}}{3x^{1/2}}$.

- (a) Show that $F = 3x^{7/2}$. [3 marks]
- (b) Find x when $F = 384$. [3 marks]

33. [Maximum mark: 5]

Let $p = \log_3 2$.

- (a) Express $\log_3 18$ in terms of p . [2 marks]
- (b) Hence express $\log_{18} 4$ in terms of p . [3 marks]

34. [Maximum mark: 5]

The number x satisfies $\log_2(x - 2) + \log_2(x + 2) = 5$.

- (a) State the restriction on x . [1 mark]
- (b) Find x . [4 marks]

35. [Maximum mark: 4]

Consider $\log_3(x + 4) - \log_3 x = 1$.

- (a) State the restriction on x . [1 mark]
- (b) Solve the equation. [3 marks]

36. [Maximum mark: 5]

The logarithm $\log_a 81$ is defined and $\log_a 81 = 4$.

- (a) Find a . [3 marks]
- (b) Hence evaluate $\log_a(1/27)$. [2 marks]

37. [Maximum mark: 4]

The real number x satisfies $8^{x-1} = 4^{2x+1}$.

- (a) Write both sides as powers of 2. [2 marks]
- (b) Hence solve for x . [2 marks]

**38.** [Maximum mark: 6]

The real number x satisfies $7^{2x-1} = 3$.

- (a) Find an exact expression for x . [3 marks]
- (b) Show that $\frac{1}{2} < x < 1$. [3 marks]

39. [Maximum mark: 6]

Let $u = 2^x$.

- (a) Show that $4^x - 6 \cdot 2^x + 8 = 0$ can be written as $u^2 - 6u + 8 = 0$. [2 marks]
- (b) Hence find all real solutions of $4^x - 6 \cdot 2^x + 8 = 0$. [4 marks]

40. [Maximum mark: 4]

The equation $\log_2 x + \log_8 x = 4$ has a positive solution.

- (a) Express both logarithms using base 2. [2 marks]
- (b) Hence find x . [2 marks]

41. [Maximum mark: 6]

An exponential model is $N(t) = A \cdot 2^{kt}$, where $A > 0$ and $k > 0$. The table gives two values.

t	0	3
$N(t)$	10	80

- (a) Find A and k . [4 marks]
- (b) Find the exact value of t for which $N(t) = 160$. [2 marks]

42. [Maximum mark: 5]

Two quantities are modelled by $A(t) = 4 \cdot 3^t$ and $B(t) = 12 \cdot 2^t$.

- (a) Show that $A(t) = B(t)$ implies $(3/2)^t = 3$. [2 marks]
- (b) Hence find the exact value of t when the quantities are equal. [3 marks]

43. [Maximum mark: 5]

Let $a > 0$, $a \neq 1$, and $x, y > 0$.

- (a) Write $\log_a \sqrt{x} - 2 \log_a y$ as a single logarithm. [3 marks]
- (b) Find x in terms of y when this expression is zero. [2 marks]

44. [Maximum mark: 5]

Let $L = \log_2 9 \log_3 4$.

- (a) Use change of base to show that $L = 4$. [3 marks]
- (b) Hence solve $2^{xL} = 32$. [2 marks]

**45.** [*Maximum mark: 6*]

Consider $2 \ln x = \ln(5x - 4)$.

- (a) State the restriction on x . [2 marks]
- (b) Solve the equation. [4 marks]

46. [*Maximum mark: 5*]

Let $A = \log_5 75 - \log_5 3$.

- (a) Find A . [2 marks]
- (b) Hence solve $25^{x-1} = 5^{Ax+1}$, or explain why there is no solution. [3 marks]

47. [*Maximum mark: 5*]

For $x > 1$, let $H(x) = \log_4(x - 1) + \log_2 \sqrt{x - 1}$.

- (a) Show that $H(x) = \log_2(x - 1)$. [3 marks]
- (b) Hence solve $H(x) = 3$. [2 marks]

48. [*Maximum mark: 5*]

The real number x satisfies $5^{x+1} = 2^{2x-1}$.

- (a) Show that $(5/4)^x = 1/10$. [2 marks]
- (b) Hence find an exact expression for x and state whether it is positive or negative. [3 marks]

49. [*Maximum mark: 5*]

For $x > 0$, define $E = \ln(9x^2) - 2 \ln(3x)$.

- (a) Simplify E . [3 marks]
- (b) Hence solve $e^{x+E} = e^3$. [2 marks]

50. [*Maximum mark: 6*]

A model is $V(t) = 160 \cdot 2^{-t/3}$, where t is in hours and V is in millilitres.

- (a) Find the time when $V = 20$. [3 marks]
- (b) Find an exact expression for the time when $V = 50$. [3 marks]

4 AA HL exclusive challenge

For HL students: deeper problems using the same SL 1.7 content. Non-calculator.

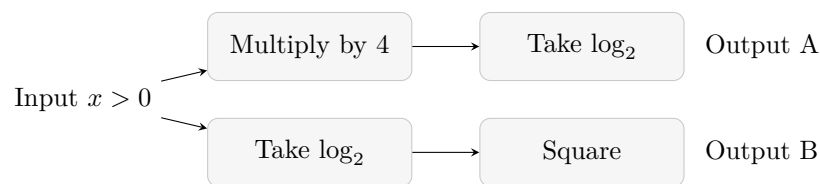
51. Solve $\log_2(x-1) + \log_4((x+3)^2) = 3$, giving all real solutions and checking the logarithm arguments.

52. Find all positive real values of x satisfying $x^{\log_2 x} = 16$.

53. For a real parameter k , consider $4^x - k2^x + 4 = 0$. Determine all values of k for which there are exactly two distinct real solutions, and find the sum of those solutions.

54. Let $a, b > 0$, with $a, b \neq 1$, and suppose $\log_a b = 2 \log_b a$. Find every possible value of $\log_a b$ and express b in terms of a .

55. The same positive input x is sent through the two routes below. Find all inputs for which the outputs are equal.





56. Solve $\log_x 16 + \log_{4x} 16 = 3$ over the reals. State all restrictions on the bases.

57. An exponential model is $N(t) = Ab^t$, with $A, b > 0$. Two exact observations are shown.

t (hours)	2	8
$N(t)$	3	648

(a) Find A and b without using a calculator.

(b) Find the exact time when $N(t) = 81$.

58. Let $a, b > 0$, with $a, b \neq 1$ and $ab \neq 1$. Set $p = \log_a b$ and $q = \log_{ab}(a/b)$.

(a) Express q in terms of p .

(b) Find p when $q = 3$. Explain why this is compatible with positive logarithm bases.

59. Let $a > 0$, $a \neq 1$. Determine all real solutions of $a^{2x} - 5a^x + 4 = 0$. Then find all values of a for which the two solutions differ by 2.

60. Let k be real. Determine how many real solutions the equation $\ln(x-1) + \ln(x+1) = \ln(kx)$ has for each value of k . Give the solution whenever one exists.

**Answer key and solution outlines**

Exact equivalents are accepted. For GDC questions, decimals below are rounded only at the final step.

Section 1: Short response**1.**

(a) $81^{3/4} = 3^3 = 27$.

(b) $64^{-2/3} = 1/4^2 = 1/16$.

(c) $(16/81)^{-3/4} = (2/3)^{-3} = 27/8$.

2.

(a) $a^{5/6}$.

(b) $a^{-2/3}$.

3.

(a) $x^{5/6+1/3-1/2} = x^{2/3}$.

(b) $8^{2/3}x^4 = 4x^4$.

4.

(a) 2, since $7^2 = 49$.

(b) -3, since $5^{-3} = 1/125$.

(c) $1/2$, since $\sqrt{e} = e^{1/2}$.

5.

(a) $\log_3 27 = 3$.

(b) $\log_5 125 = 3$.

6. $\ln 5 + 3 \ln x - \frac{1}{2} \ln y$.

7. Use $2 = \log_2 4$: $\log_2 \left(\frac{4x^3}{y} \right)$.

8.

(a) $3/2$, since $9^{3/2} = 27$.

(b) $-3/2$, since $(2^{-2})^{-3/2} = 8$.

9.

(a) $\ln 12 / \ln 7$.

(b) $\ln 2 / \ln 5 = \log_5 2$.

10. $2^{2x+2} = 2^5$, so $x = 3/2$.

11. $3^{4x-2} = 3^{3x+3}$, so $x = 5$.

12. $x \ln 5 = \ln 14$, so $x = \ln 14 / \ln 5$.

13. $3x - 2 = \ln 7$, so $x = (2 + \ln 7)/3$.

**Section 1: Short response (continued)****14.**

(a) $2x - 5 > 0 \Rightarrow x > 5/2.$

(b) $7 - x > 0 \Rightarrow x < 7.$

15. The product law does not apply to a sum. The left side is $\log_{10} 10 = 1$; the right side is $\log_{10} 16$, which is not 1.

Section 2: Extended response**16.**

(a) $A = 3^{-2} = 1/9$; $B = 3^{-1/2} = 1/\sqrt{3}$; $C = 3^{3/2} = 3\sqrt{3}.$

(b) $1/\sqrt{27} = 3^{-3/2}$, so $x = -3/2.$

17.

(a) $E = x^{2/3+1/2-1/6} = x^1 = x.$

(b) $x = 81^{1/4} = 3.$

18.

(a) $\log_2(3^2 \cdot 5) = 2p + q.$

(b) $\log_6 25 = \frac{\log_2 25}{\log_2 6} = \frac{2q}{1+p}.$

19.

(a) $x > 1$ and $\log_3(x^2 - 1) = 2.$

(b) $x^2 - 1 = 9 \Rightarrow x = \pm\sqrt{10}$. Only $x = \sqrt{10}$ satisfies $x > 1.$

20.

(a) $x > 0$ (which also ensures $x + 6 > 0$).

(b) $\ln((x+6)/x) = \ln 4 \Rightarrow x+6 = 4x \Rightarrow x = 2.$

21. Domain: $x > 5/6$. Then $x^2 = 6x - 5 \Rightarrow (x-1)(x-5) = 0$. Both $x = 1$ and $x = 5$ are valid.

22.

(a) $u^2 - 10u + 9 = (u-1)(u-9) = 0.$

(b) $3^x = 1$ or 9 , so $x = 0$ or $2.$

23.

(a) $(x+2)\ln 2 = (x-1)\ln 5$, so $x(\ln 5 - \ln 2) = \ln 5 + 2\ln 2.$

(b) $x = \frac{\ln 20}{\ln(5/2)}.$

24.

(a) $\ln 17 / \ln 6 = 1.58$ (3 s.f.).

(b) $\ln 7 / \ln 0.4 = -2.12$ (3 s.f.).

**Section 2: Extended response (continued)****25.**

- (a) $600(1.18)^5 = 1372.65\dots$, so 1373 organisms.
(b) $1.18^t = 2.5 \Rightarrow t = \ln 2.5 / \ln 1.18 = 5.54$ hours (3 s.f.).

26.

- (a) $e^{-0.12t} = 1/2$, so $t = \ln 2 / 0.12 = 5.78$ days.
(b) $e^{-0.12t} = 1/8$, so $t = \ln 8 / 0.12 = 17.3$ days: exactly three half-lives.

27.

- (a) $A = 5$ from $t = 0$; $5b^2 = 45$ gives $b = 3$.
(b) $3^t = 4 \Rightarrow t = \ln 4 / \ln 3$.

28.

- (a) $\log_4 x = \log_2 x / 2$.
(b) $\frac{3}{2} \log_2 x = 6 \Rightarrow \log_2 x = 4 \Rightarrow x = 16$.

29.

- (a) $2^2 = 4 < 6 < 8 = 2^3$ and 2^x increases, so $2 < \alpha < 3$.
(b) $2^\alpha = 6 \Rightarrow \alpha = \ln 6 / \ln 2$.

30.

- (a) Both logarithm arguments must be positive: $x > 2$.
(b) $x^2 - 4 = 5 \Rightarrow x = \pm 3$; reject -3 , so $x = 3$ only.

Section 3: IB exam-style / Paper 1**31.**

- (a) $a = 4$, $b = 1/4$.
(b) $ab = 1$, so $(ab)^{3/2} = 1$.

32.

- (a) $F = 9x^4 / (3x^{1/2}) = 3x^{7/2}$.
(b) $x^{7/2} = 128 = 2^7$, so $x = 4$.

33.

- (a) $\log_3(2 \cdot 3^2) = p + 2$.
(b) $\log_{18} 4 = \frac{\log_3 4}{\log_3 18} = \frac{2p}{p+2}$.

34.

- (a) $x > 2$.
(b) $(x-2)(x+2) = 32 \Rightarrow x^2 = 36$. Only $x = 6$ is allowed.



Section 3: IB exam-style / Paper 1 (continued)

35.

- (a) $x > 0$.
(b) $(x + 4)/x = 3 \Rightarrow x + 4 = 3x \Rightarrow x = 2$.

36.

- (a) $a^4 = 81$ and $a > 0$, so $a = 3$.
(b) $\log_3(3^{-3}) = -3$.

37.

- (a) $2^{3x-3} = 2^{4x+2}$.
(b) $3x - 3 = 4x + 2 \Rightarrow x = -5$.

38.

- (a) $(2x - 1) \ln 7 = \ln 3 \Rightarrow x = \frac{1}{2}(1 + \ln 3 / \ln 7)$.
(b) $1 < 3 < 7 \Rightarrow 0 < \log_7 3 < 1$. Thus $0 < 2x - 1 < 1$ and $1/2 < x < 1$.

39.

- (a) $4^x = (2^x)^2 = u^2$, giving the stated equation.
(b) $(u - 2)(u - 4) = 0$, so $2^x = 2$ or 4 and $x = 1$ or 2 .

40.

- (a) $\log_8 x = \frac{1}{3} \log_2 x$, so $\log_2 x + \frac{1}{3} \log_2 x = 4$.
(b) $\log_2 x = 3 \Rightarrow x = 8$.

41.

- (a) $A = 10$; $10 \cdot 2^{3k} = 80 \Rightarrow 3k = 3 \Rightarrow k = 1$.
(b) $10 \cdot 2^t = 160 \Rightarrow 2^t = 16 \Rightarrow t = 4$.

42.

- (a) $4 \cdot 3^t = 12 \cdot 2^t \Rightarrow 3^t / 2^t = 3 \Rightarrow (3/2)^t = 3$.
(b) $t \ln(3/2) = \ln 3 \Rightarrow t = \ln 3 / \ln(3/2)$.

43.

- (a) $\log_a(\sqrt{x}/y^2)$.
(b) $\sqrt{x}/y^2 = a^0 = 1 \Rightarrow x = y^4$.

44.

- (a) $L = \frac{2 \ln 3}{\ln 2} \cdot \frac{2 \ln 2}{\ln 3} = 4$.
(b) $2^{4x} = 2^5 \Rightarrow x = 5/4$.

45.

- (a) $x > 0$ and $5x - 4 > 0$, hence $x > 4/5$.
(b) $x^2 = 5x - 4 \Rightarrow (x - 1)(x - 4) = 0$. Both $x = 1$ and $x = 4$ are valid.

**Section 3: IB exam-style / Paper 1 (continued)****46.**

- (a) $A = \log_5 25 = 2$.
- (b) $5^{2x-2} = 5^{2x+1}$ would require $-2 = 1$. There is no real solution.

47.

- (a) Each term is $\frac{1}{2} \log_2(x-1)$, so their sum is $\log_2(x-1)$.
- (b) $x-1 = 2^3 = 8 \Rightarrow x = 9$, which satisfies $x > 1$.

48.

- (a) $5 \cdot 5^x = 4^x/2 \Rightarrow (5/4)^x = 1/10$.
- (b) $x = \ln(1/10)/\ln(5/4) = -\ln 10/\ln(5/4) < 0$.

49.

- (a) $E = \ln 9 + 2 \ln x - 2 \ln 3 - 2 \ln x = 0$.
- (b) $e^x = e^3 \Rightarrow x = 3$, consistent with $x > 0$.

50.

- (a) $2^{-t/3} = 1/8 = 2^{-3} \Rightarrow t = 9$ hours.
- (b) $2^{-t/3} = 5/16 \Rightarrow t = 3 \ln(16/5)/\ln 2$ hours.

**Section 4: AA HL exclusive challenge**

51. Domain: $x > 1$, so $x + 3 > 0$. Therefore $\log_4((x+3)^2) = \log_2(x+3)$. Then $(x-1)(x+3) = 8$, giving $x^2 + 2x - 11 = 0$. The candidates are $-1 \pm 2\sqrt{3}$; only $x = -1 + 2\sqrt{3}$ is valid.

52. Let $u = \log_2 x$, so $x = 2^u$. Then $(2^u)^u = 2^4 \Rightarrow u^2 = 4$. Thus $u = \pm 2$, giving $x = 4$ or $x = 1/4$. Both satisfy the original equation.

53. Set $u = 2^x > 0$. The quadratic $u^2 - ku + 4 = 0$ must have two distinct positive roots. Since their product is 4 and sum is k , this requires $k > 4$. If the roots are u_1, u_2 , then $x_1 + x_2 = \log_2(u_1 u_2) = \log_2 4 = 2$.

54. Set $r = \log_a b \neq 0$. By change of base, $\log_b a = 1/r$, so $r = 2/r$ and $r = \pm\sqrt{2}$. Hence $b = a^{\sqrt{2}}$ or $b = a^{-\sqrt{2}}$. Both are permitted for any $a > 0, a \neq 1$.

55. $\log_2(4x) = (\log_2 x)^2$. Set $u = \log_2 x$ to obtain $u + 2 = u^2$, so $(u-2)(u+1) = 0$. Hence $x = 4$ or $x = 1/2$.

56. $x > 0, x \neq 1, x \neq 1/4$. Let $u = \log_2 x$; then $u \neq 0, -2$ and $4/u + 4/(u+2) = 3$. Multiplying gives $3u^2 - 2u - 8 = (3u+4)(u-2) = 0$. Hence $x = 4$ or $x = 2^{-4/3}$; both meet the base restrictions.

57.

(a) Dividing the observations gives $b^6 = 216 = 6^3$, hence $b = \sqrt{6}$. Then $Ab^2 = 3 \Rightarrow A = 1/2$.

(b) $\frac{1}{2}(\sqrt{6})^t = 81 \Rightarrow 6^{t/2} = 162$, so $t = 2 \ln 162 / \ln 6$.

58.

(a) $q = \frac{\ln a - \ln b}{\ln a + \ln b} = \frac{1-p}{1+p}$, with $p \neq -1$.

(b) $3(1+p) = 1-p \Rightarrow p = -1/2$. Thus $b = a^{-1/2} > 0$; a and b lie on opposite sides of 1, and $ab = \sqrt{a} \neq 1$.

59. Let $u = a^x > 0$. Then $(u-1)(u-4) = 0$, so $x = 0$ or $x = \ln 4 / \ln a$. Their distance is $|\ln 4 / \ln a|$. Setting this to 2 gives $\ln a = \pm \ln 2$, so $a = 2$ or $a = 1/2$.

60. Domain: $x > 1$ and $kx > 0$, so necessarily $k > 0$. If $k \leq 0$, there is no real solution. If $k > 0$, $x^2 - 1 = kx$ gives $x = (k \pm \sqrt{k^2 + 4})/2$. The minus root is negative; the plus root exceeds 1 because $k + \sqrt{k^2 + 4} > 2$. Thus exactly one solution exists for $k > 0$: $x = (k + \sqrt{k^2 + 4})/2$.