



Transformations of Graphs

IB Mathematics AA SL & HL | Worksheet 2.11

50 shared questions + 10 HL challenges

At a glance

Learning goal	Describe transformations; map points; sketch graphs; track domain, range and asymptotes.
Syllabus link	Shared: SL 2.11. HL extension: AHL 2.16, restricted here to combined inputs $f(ax + b)$.
Structure	1–15 short response; 16–30 extended response; 31–50 IB-style; 51–60 HL exclusive.
Technology	Non-calculator except 25(g) and 29(e), marked GDC . The Paper 1 section is entirely non-calculator.

Instructions

Work on separate paper and use the supplied axes for sketches. Give exact answers unless an approximation is requested. Label key features. **SF** means scale factor. State a valid *ordered sequence* when requested; more than one sequence may be correct. Use **stretch** with its scale factor. Unless stated otherwise, use the natural real domain. Questions 1–50 are shared SL/HL practice; Questions 51–60 extend the syllabus for HL.

1. Short response

Identify the change to the input or output, and track the corresponding coordinates.

1. For each equation, name the transformation, state its direction or scale factor, and classify it as horizontal or vertical.

- (a) $y = f(x) + 6$.
- (b) $y = f(x - 4)$.
- (c) $y = \frac{1}{3}f(x)$.

2. Describe each transformation fully.

- (a) $y = f(5x)$.
- (b) $y = f(x/4)$.
- (c) $y = -f(x)$.
- (d) $y = f(-x)$.

3. The point $P = (-2, 7)$ lies on $y = f(x)$. Find its corresponding point on each graph.

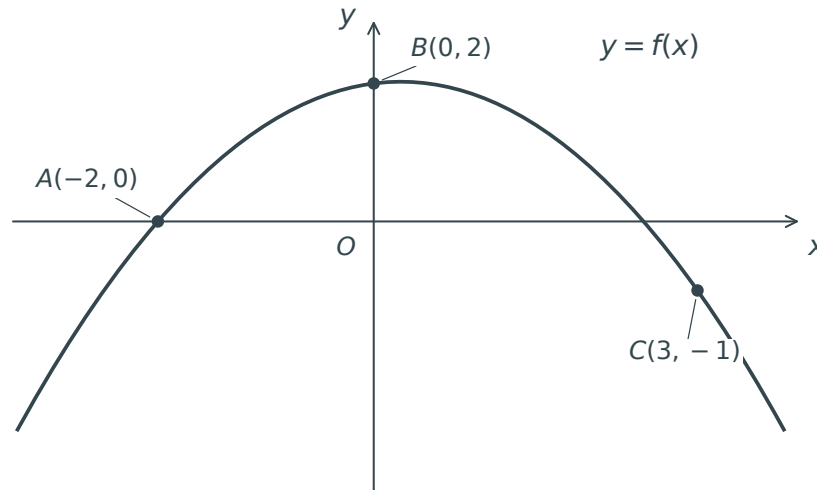
- (a) $y = f(x) - 3$.
- (b) $y = f(x + 5)$.
- (c) $y = 2f(x)$.
- (d) $y = f(2x)$.



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4. Write the point mapping for each transformation, using a generic point (x, y) .
- (a) $y = -f(x)$.
 - (b) $y = f(-x)$.
 - (c) $y = f(x/3)$.
-
5. The point $A = (8, -3)$ lies on $y = f(x)$. Its corresponding point on $y = f(kx)$ is $A' = (2, -3)$, where $k > 0$.
- (a) Find k .
 - (b) State the horizontal scale factor.
-
6. The only roots of f are $x = -3$ and $x = 5$.
- (a) State the roots of $y = 4f(x)$.
 - (b) State the roots of $y = -f(x)$.
 - (c) Explain why these transformations preserve the original roots.
-
7. The domain of f is $[-5, 2)$ and its range is $(-1, 6]$. Let $g(x) = f(x) + 4$.
- (a) State the domain of g .
 - (b) State the range of g .
-
8. The domain of f is $[-5, 2)$ and its range is $(-1, 6]$. Let $g(x) = f(x - 3)$.
- (a) State the domain of g .
 - (b) State the range of g .
-
9. The range of f is $[-2, 5)$. Let $g(x) = -2f(x)$.
- (a) State the range of g .
 - (b) Explain which endpoint is included.
-
10. Write an equation for the image of $y = f(x)$ after each sequence.
- (a) Translate 2 units left, then 5 units down.
 - (b) Stretch vertically with SF 3, then reflect across the x -axis.

11. The sketch shows a function f with labelled points A , B and C .

- State the images of A , B and C on $y = 2f(x)$.
- Explain why this change is not simply a vertical translation.



12. The graph of f has asymptotes $x = -1$ and $y = 2$. It is translated 4 units right and 3 units down.

- State the new vertical asymptote.
- State the new horizontal asymptote.

13. Given $f(3) = 8$, explain why the point $(5, 8)$ lies on $y = f(x - 2)$. Use the input expression in your explanation.

14. It is given that $f(-6) = 4$.

- Find the corresponding point on $y = f(3x)$.
- Explain why the horizontal scale factor is $1/3$, rather than 3.

15. A student writes: "For $y = f(x) + 5$, every root of f moves 5 units up and remains a root.

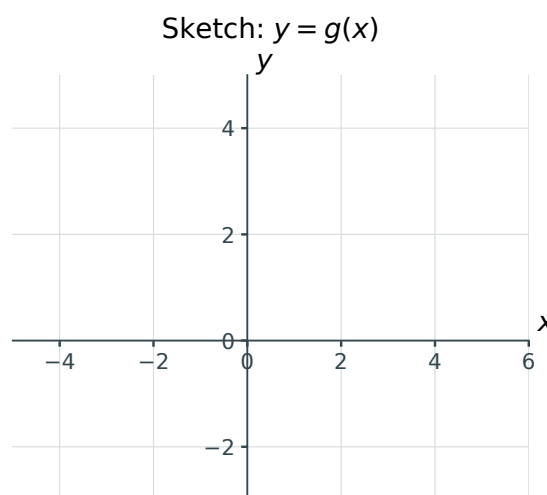
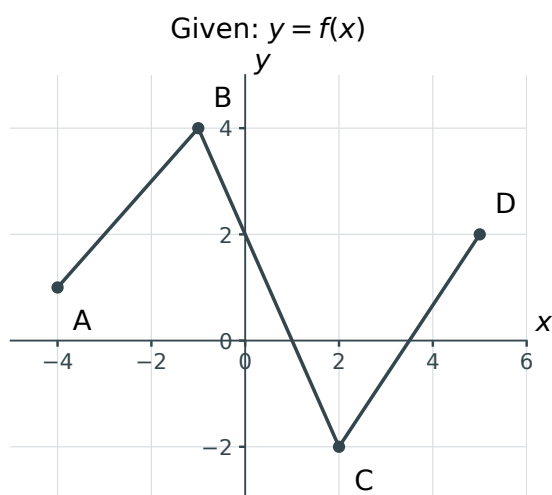
- Identify the error.
- State the condition that determines the new roots.

2. Extended response

Use the original graph to follow key points, domain and range through each transformation.

16. The graph shown consists of straight segments joining $A(-4, 1)$, $B(-1, 4)$, $C(2, -2)$ and $D(5, 2)$, including its endpoints. Let $g(x) = -f(x) + 2$.

- State the images of A, B, C, D .
- Sketch g on the supplied axes.
- State the domain of g .
- State the range of g .
- Explain why the domain is unchanged.

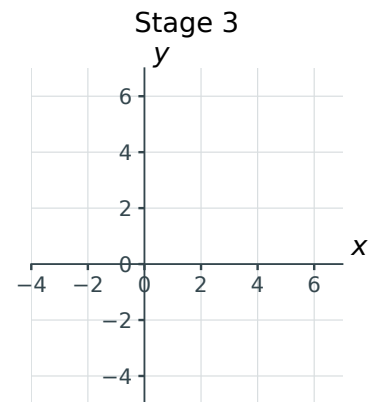
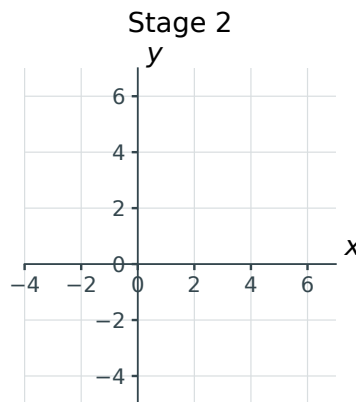
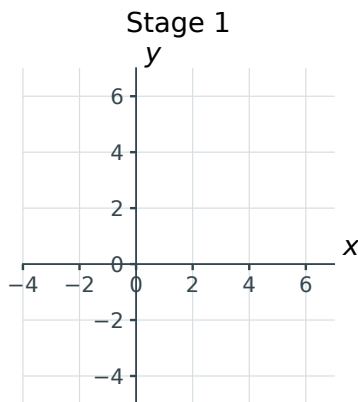


17. Let $f(x) = x^2 - 6x + 5$.

- Find and simplify $f(x - 2)$.
- State the coordinates of the original vertex and x -intercepts.
- State the images of these three points.
- State the domain of $f(x - 2)$.
- State its range.

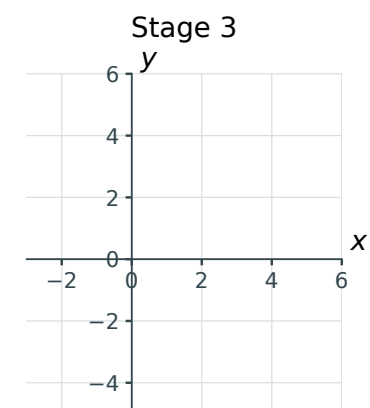
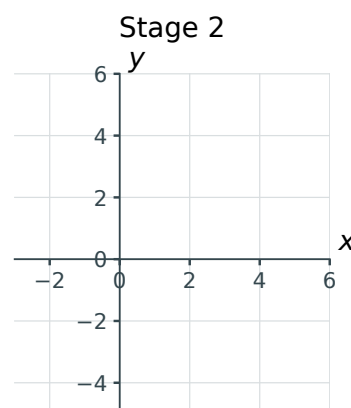
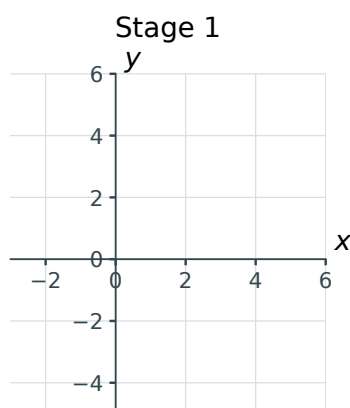
18. Starting from $y = \sqrt{x}$, obtain $y = 2\sqrt{x+3} - 4$.

- List a valid sequence of three transformations, with the horizontal change first.
- Write the equation after each transformation.
- Track the point $(1, 1)$ through the three stages.
- State the domain after each stage.
- State the range after each stage.
- Sketch the three stages on the supplied axes. Label the tracked point in each sketch.



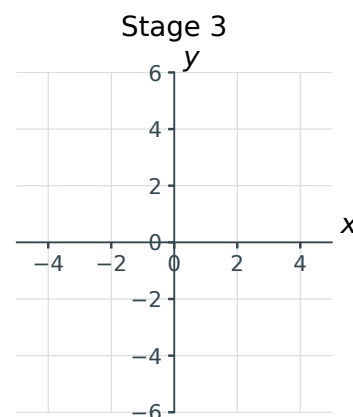
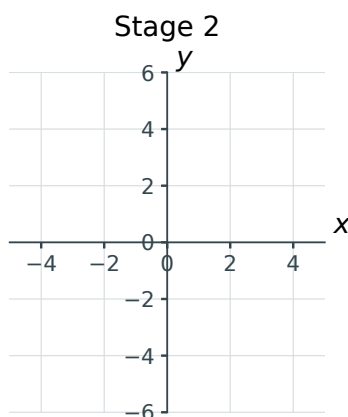
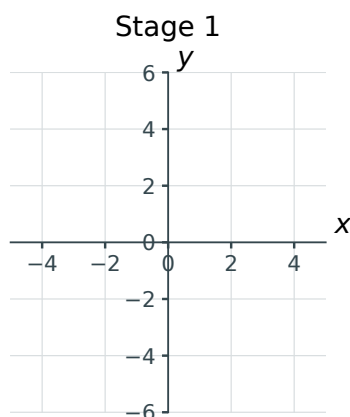
19. Starting from $y = 2^x$, obtain $y = 3 - 2^{x-2}$.

- List a valid sequence of three transformations, with the horizontal change first.
- Write the equation after each transformation.
- Track the point $(0, 1)$ through the three stages.
- State the domain after each stage.
- State the range after each stage.
- State the horizontal asymptote after each stage.
- Sketch the three stages on the supplied axes. Label the tracked point in each sketch.



20. Begin with $y = 1/x$. Apply, in order: a translation 1 unit left; a vertical stretch with scale factor 3; a translation 2 units down.

- Write the equation after each stage.
- State the domain after each stage.
- State the range after each stage.
- State both asymptotes after each stage.
- Sketch the three stages on the supplied axes.
- Express the final equation as a single fraction.



21. The domain of f is $(-2, 6]$ and its range is $[-3, 5)$. Let $g(x) = -2f(x + 4) + 1$.

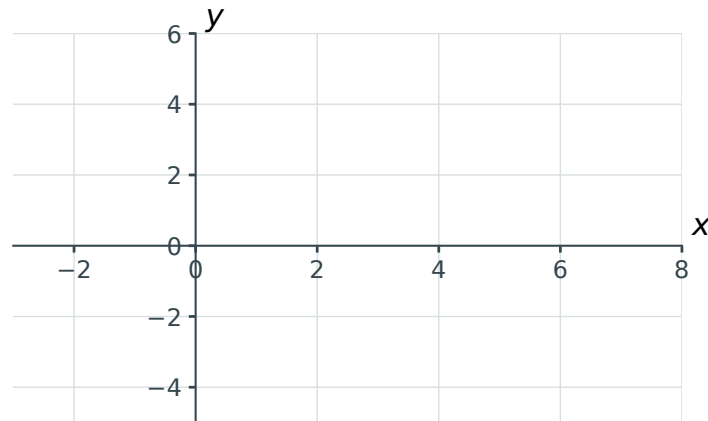
- State one valid ordered sequence of transformations.
- Find the domain of g .
- Find the range of g .
- The point $P = (2, -1)$ lies on f . Find its image.

22. For parts (a) and (b), write the equation produced by the stated sequence applied to $y = f(x)$:

- Stretch with SF 4, then translate 3 units down.
- Translate 3 units down, then stretch with SF 4.
- If the translation is to be performed first, how far down should it be to produce the result in part (a)?

23. Let $g(x) = -3/(x - 2) + 1$.

- List a valid sequence from $y = 1/x$ to $y = g(x)$.
- State the domain.
- State the range.
- State both asymptotes.
- Find both intercepts.
- Sketch g on the supplied axes.

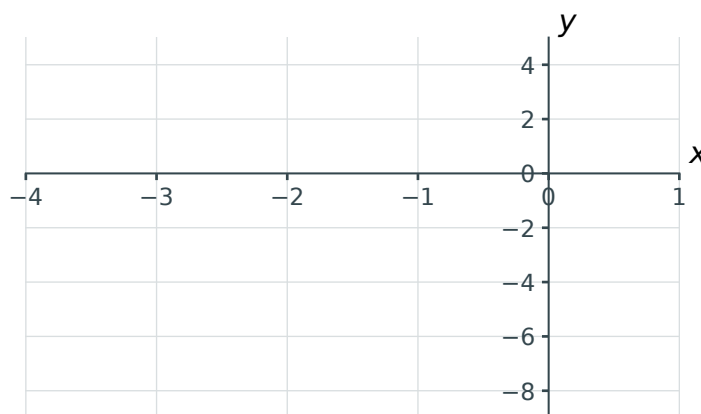


24. A function f has a minimum at $(2, -3)$. The graph of $g(x) = af(x) + d$, with $a < 0$, has a maximum at $(2, 10)$ and contains the image $(5, 2)$ of the point $(5, 1)$ on f .

- Find a and d .
- Describe the vertical transformations.
- Explain why the minimum becomes a maximum.

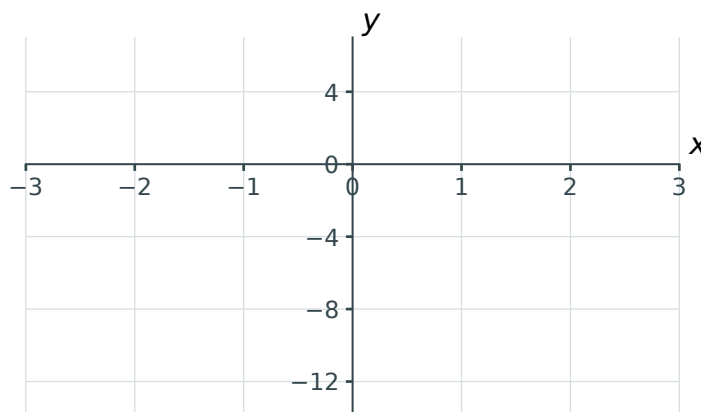
25. **GDC** Let $f(x) = x^3$ and $g(x) = -(x + 2)^3 + 1$.

- List a valid sequence from f to g .
- Find the images of $(-1, -1)$, $(0, 0)$, $(1, 1)$.
- Find both intercepts of g .
- State the domain of g .
- State its range.
- Sketch g on the supplied axes.
- GDC:** solve $g(x) = x$, giving your answer to 3 significant figures.



26. For $f(x) = x^2 - 4$, compare $g(x) = 3f(x)$ and $h(x) = f(x) + 3$.

- Find equations for g and h .
- State their vertices.
- Find their roots.
- Sketch both graphs on the supplied axes.
- Explain which transformation preserves the original roots.





27. The graph of f has range $[2, \infty)$ and contains $(1, 4)$. The graph of $g(x) = af(x) + d$, where $a < 0$, has range $(-\infty, 7]$ and contains $(1, 1)$.

- (a) Find a and d .
- (b) Give a valid ordered sequence of transformations.

28. The graph of f has domain $[-4, 8]$, range $[-2, 3]$ and contains $P = (6, 1)$. Define $g(x) = f(2x)$, $h(x) = f(x/2)$ and $k(x) = f(-x)$.

- (a) Find the domain of each of g, h, k .
- (b) Find the range of each of g, h, k .
- (c) Find the image of P on each graph.

29. **GDC** A model $H(t) = 9e^{-0.08(t-4)^2}$ gives the height, in metres, of a moving object for $0 \leq t \leq 12$ seconds. Its maximum is 9 metres at $t = 4$. Define $K(t) = \frac{1}{2}H(t/2) + 1$.

- (a) Describe the transformations from H to K .
- (b) State the domain of K .
- (c) Find the maximum height predicted by K .
- (d) Find the time when this maximum occurs.
- (e) **GDC:** find the two times when $K(t) = 4$. Give answers to 3 significant figures.

30. The following table gives some values of f .

x	-2	0	3	7
$f(x)$	5	-1	4	0

Let $g(x) = -2f(x - 1) + 3$.

- (a) Make a table of the corresponding inputs and outputs of g .
- (b) Evaluate $g(4)$.
- (c) Explain why the table alone does not determine the full domain or range of g .

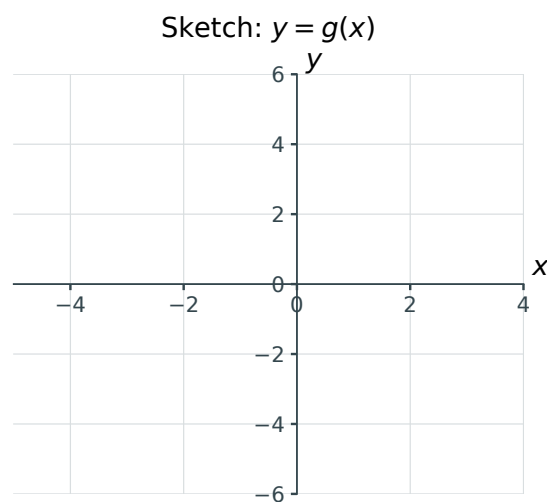
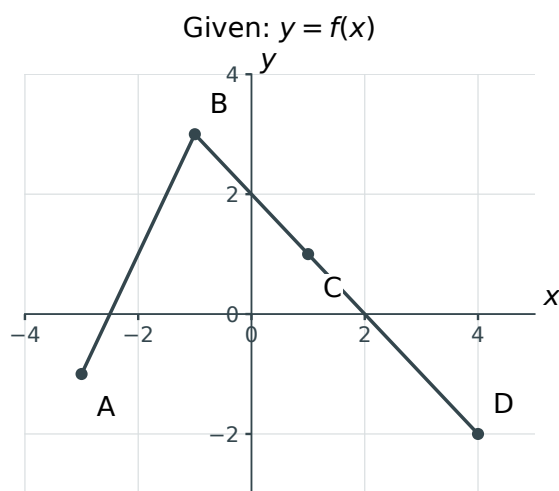
3. IB exam-style questions | Paper 1

These are original practice questions. The maximum mark and marks for each part are shown.

31. [Maximum mark: 9]

The graph of f joins $A(-3, -1)$, $B(-1, 3)$, $C(1, 1)$ and $D(4, -2)$ by straight segments, including the endpoints. Let $g(x) = 2f(x+1) - 1$.

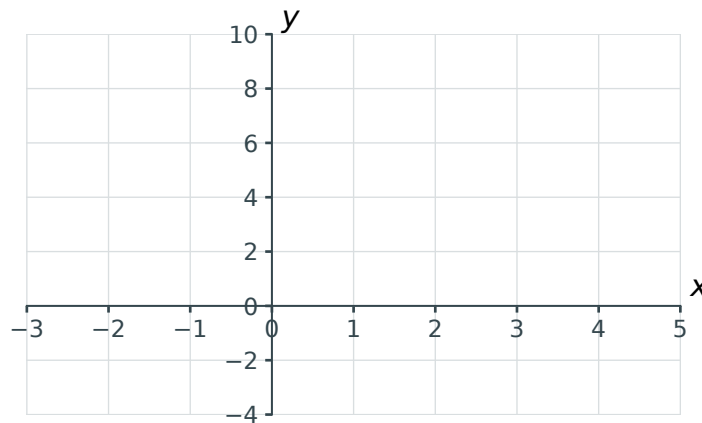
- Describe a valid ordered sequence from f to g . [3]
- Find the images of A, B, C, D . [2]
- Sketch g on the supplied axes. [2]
- State the domain of g . [1]
- State the range of g . [1]



32. [Maximum mark: 10]

Let $g(x) = -2(x - 1)^2 + 8$.

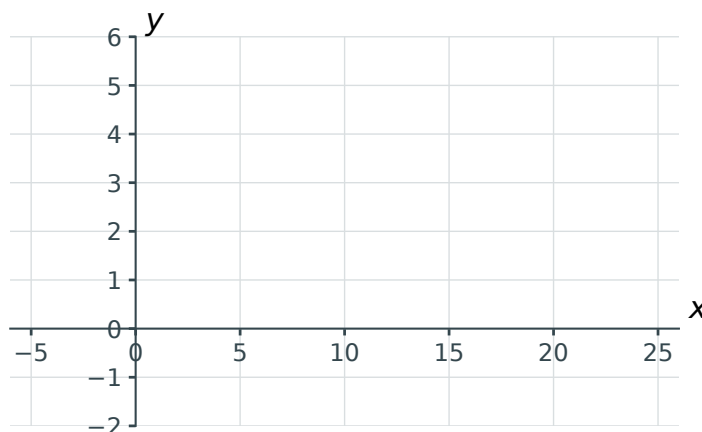
- List a valid ordered sequence from $y = x^2$ to $y = g(x)$. [3]
- State the vertex. [1]
- Find both x -intercepts. [2]
- Find the y -intercept. [1]
- State the range. [1]
- Sketch g on the supplied axes. [2]



33. [Maximum mark: 10]

The graph of $g(x) = 5 - \sqrt{x+4}$ is obtained from $y = \sqrt{x}$.

- State a valid ordered transformation sequence. [3]
- State the endpoint of g . [1]
- State its domain. [1]
- State its range. [1]
- Find both intercepts. [2]
- Sketch g on the supplied axes. [2]



34. [Maximum mark: 9]

Let $g(x) = a3^x + d$, where $a < 0$. Its horizontal asymptote is $y = 4$, and its graph passes through $(1, -2)$.

- Find a and d . [3]
- Describe a valid sequence from $y = 3^x$ to $y = g(x)$. [3]
- State the domain of g . [1]
- State its range. [1]
- Find its y -intercept. [1]

35. [Maximum mark: 9]

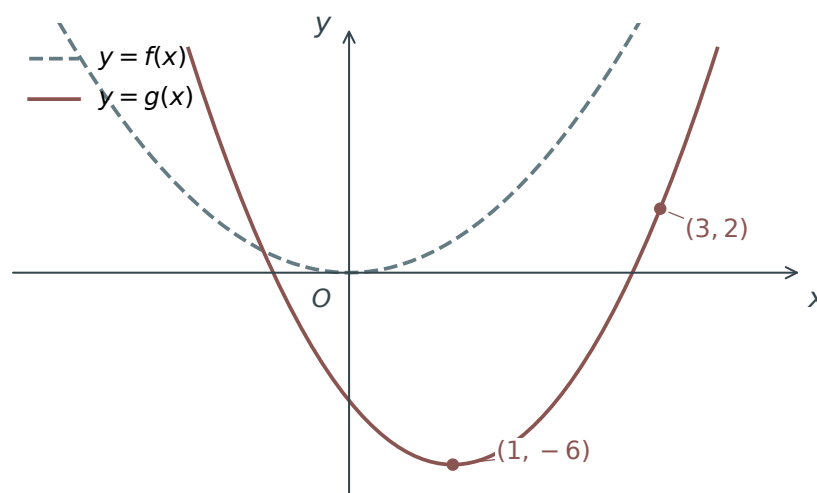
Let $g(x) = a/(x - h) + k$. Its asymptotes are $x = 3$ and $y = -2$, and its graph passes through $(4, 2)$.

- Find h, k, a . [3]
- Describe a valid sequence from $y = 1/x$ to $y = g(x)$. [3]
- Find the x -intercept. [1]
- State the domain. [1]
- State the range. [1]

36. [Maximum mark: 9]

The sketch shows $y = f(x)$ and $y = g(x)$, where $f(x) = x^2$ and $g(x) = a(x - h)^2 + k$. The point $(3, 2)$ lies on g .

- Write down h and k . [2]
- Find a . [1]
- List a valid transformation sequence from f to g . [3]
- State the range of g . [1]
- Find its roots. [2]



**37. [Maximum mark: 7]**

A graph $y = f(x)$ is stretched vertically with SF 3, then translated 8 units upwards to give $y = g(x)$.

- (a) Express $g(x)$ in terms of $f(x)$. [1]
- (b) Find the translation needed if it is performed before the same vertical stretch. [2]
- (c) Track the point $(2, -1)$ through the original sequence. [2]
- (d) Track the same point through your sequence from part (b). [2]

38. [Maximum mark: 7]

Let $f(x) = 2^x$ and $g(x) = 2^{x+3}$.

- (a) Describe a single horizontal transformation taking f to g . [1]
- (b) Show that a single vertical stretch also produces g , stating its scale factor. [2]
- (c) Find the image of $(0, 1)$ under the horizontal transformation. [1]
- (d) Find its image under the vertical transformation. [1]
- (e) Explain why the image points differ although the final graph is the same. [2]

39. [Maximum mark: 8]

The domain of f is $[-1, 4)$ and its range is $(2, 9]$. Define $g(x) = -\frac{1}{2}f(x-3) + 6$.

- (a) State a valid ordered transformation sequence. [3]
- (b) Find the domain of g , retaining the correct endpoints. [2]
- (c) Find its range, justifying endpoint inclusion. [2]
- (d) Given $f(2) = 6$, find the corresponding point on g . [1]

40. [Maximum mark: 5]

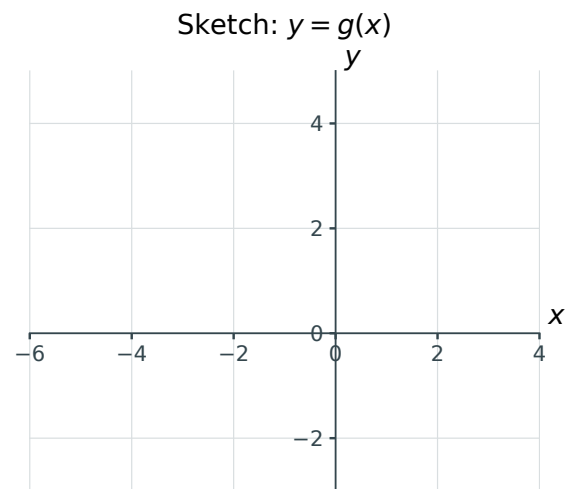
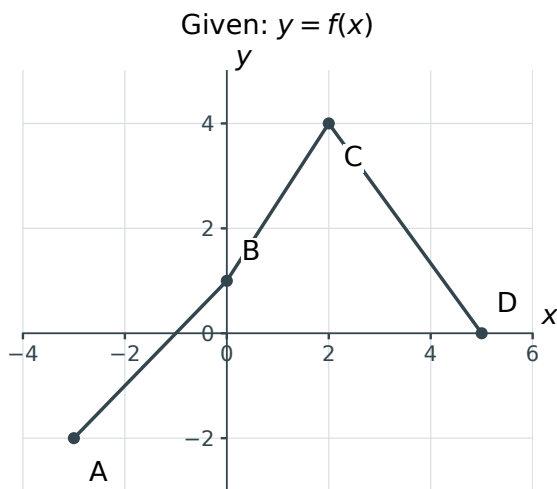
The only solutions of $f(x) = 0$ are $x = -4$ and $x = 2$. Let $g(x) = f(x+3)$.

- (a) Find the roots of g . [2]
- (b) Given $f(-1) = 5$, find the corresponding point on g . [1]
- (c) A second graph is $h(x) = f(x) + 3$. Explain why the roots of h cannot be found from the given information alone. [2]

41. [Maximum mark: 8]

The graph joins $A(-3, -2)$, $B(0, 1)$, $C(2, 4)$ and $D(5, 0)$ by straight segments, including the endpoints. Let $g(x) = f(-x)$.

- Describe the transformation. [1]
- Find the images of the four labelled points. [2]
- Sketch g on the supplied axes. [2]
- State its domain. [1]
- State its range. [1]
- The original graph has a root at $x = -1$. State the corresponding root of g . [1]



42. [Maximum mark: 10]

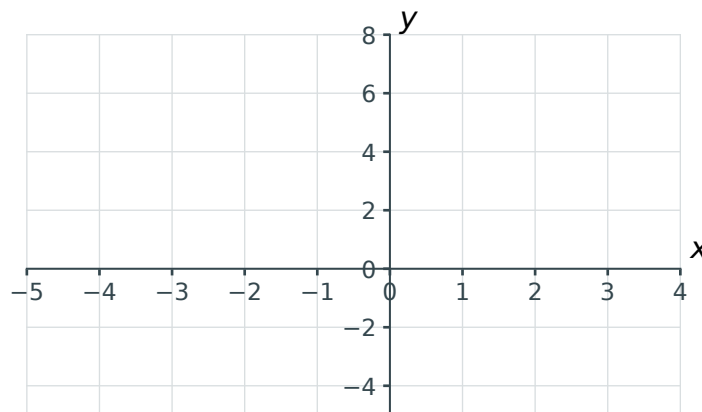
The graph of $g(x) = a\sqrt{x-h} + k$, with $a > 0$, has endpoint $(2, -3)$ and passes through $(6, 5)$.

- Write down h and k . [2]
- Find a . [1]
- Describe a valid sequence from $y = \sqrt{x}$ to $y = g(x)$. [3]
- Find the root of g . [2]
- State its domain. [1]
- State its range. [1]

43. [Maximum mark: 13]

Let $g(x) = (2x + 5)/(x + 1)$.

- Show that $g(x) = 2 + 3/(x + 1)$. [2]
- Hence describe a valid transformation sequence from $y = 1/x$. [3]
- State the domain. [1]
- State the range. [1]
- State both asymptotes. [2]
- Find both intercepts. [2]
- Sketch g on the supplied axes. [2]



44. [Maximum mark: 8]

A temperature model $T(t)$ is defined for $0 \leq t \leq 8$ hours and has range $[10, 22]$ degrees Celsius. Its maximum occurs at $t = 3$. A second model is $S(t) = \frac{1}{2}T(t/3) + 4$.

- Describe the three transformations. [3]
- State the domain of S , with units. [1]
- State its range, with units. [2]
- Find when S reaches its maximum, explaining how the input expression determines this. [2]

45. [Maximum mark: 8]

Let $g(x) = 2f(x - 4) - 3$. It is known that $g(1) = 7$ and $g(6) = -5$.

- Find $f(-3)$. [2]
- Find $f(2)$. [2]
- The range of g is $[-7, 9]$. Find the range of f . [2]
- The domain of g is $[0, 10]$. Find the domain of f . [2]

46. [Maximum mark: 9]

A function f has domain $[-2, 7]$ and range $[-4, 1]$. Its graph has a maximum at $(3, 1)$. Let $g(x) = -3f(-x) + 2$.

- List a valid ordered transformation sequence. [3]
- State the domain of g . [1]
- State its range. [2]
- Find the corresponding turning point. [1]
- Explain whether this point is a maximum or a minimum. [2]

47. [Maximum mark: 6]

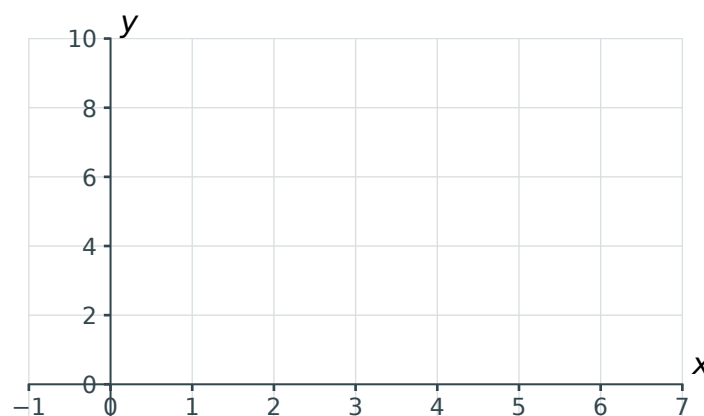
Let $f(x) = x^2 - 2x - 3$. The graph is reflected across both coordinate axes.

- Find the final equation when the x -axis reflection is performed first. [2]
- Find the final equation in the reverse order. [2]
- Explain, using (x, y) , why these two reflections commute for any function. [2]

48. [Maximum mark: 9]

The parabola $f(x) = x^2 - 6x + 5$ is translated vertically by d to produce g .

- Write $g(x)$ in vertex form. [2]
- Find d if the translated graph touches the x -axis. [2]
- State the values of d for which g has no real roots. [1]
- For $d = 4$, state the vertex. [1]
- For $d = 4$, state the range. [1]
- For $d = 4$, sketch g on the supplied axes. [2]


49. [Maximum mark: 6]

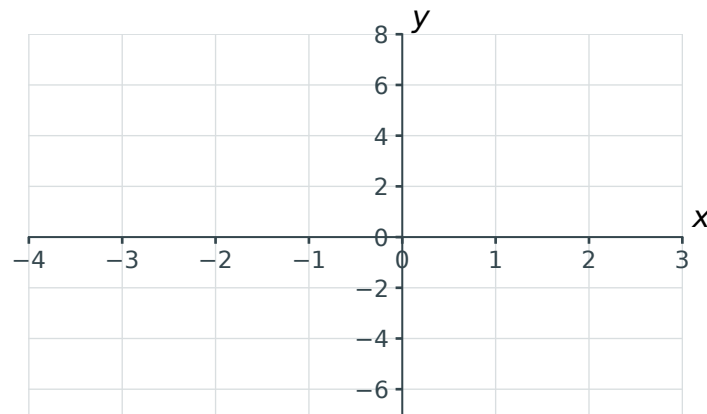
A graph is stretched vertically with SF $p > 0$ and translated upwards by $d > 0$.

- Write the final equation when the stretch is performed first. [1]
- Write the final equation in the reverse order. [1]
- Find the condition on p for these graphs to be identical for every function f . [2]
- Explain why reflection across the x -axis and a vertical stretch can always be interchanged. [2]

50. [Maximum mark: 11]

Let $g(x) = 6 - 3 \cdot 2^{x+1}$.

- (a) List a valid ordered sequence from $y = 2^x$ to $y = g(x)$. [3]
- (b) State the domain after every transformation in your sequence. [1]
- (c) State the range after every transformation. [2]
- (d) State the horizontal asymptote after every transformation. [1]
- (e) Find both intercepts of g . [2]
- (f) Sketch g on the supplied axes. [2]



4. AA HL exclusive challenge

SL students: the shared questions end at Question 50. This extension uses combined inputs $f(ax + b)$ from **AHL 2.16**. It is also useful preparation for mathematical modelling and the IA.

51. [Maximum mark: 7]

Let $g(x) = f(3x - 6)$.

- Factor the input $3x - 6$. [1]
- List a valid sequence from f to g , leaving the horizontal translation last. [2]
- Find the image of $P = (9, -2)$. [2]
- Give a second sequence with the translation first. [2]

52. [Maximum mark: 7]

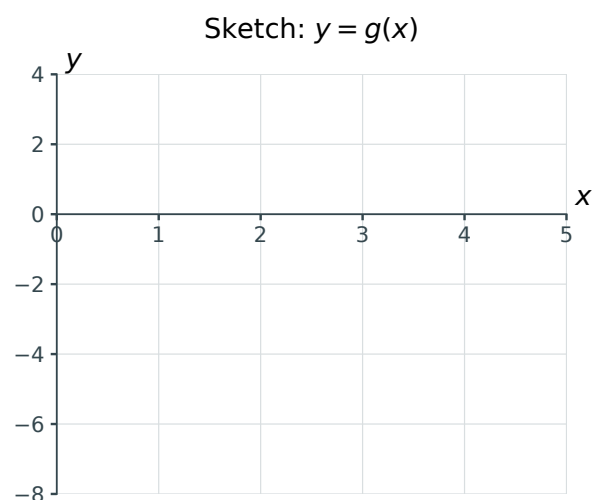
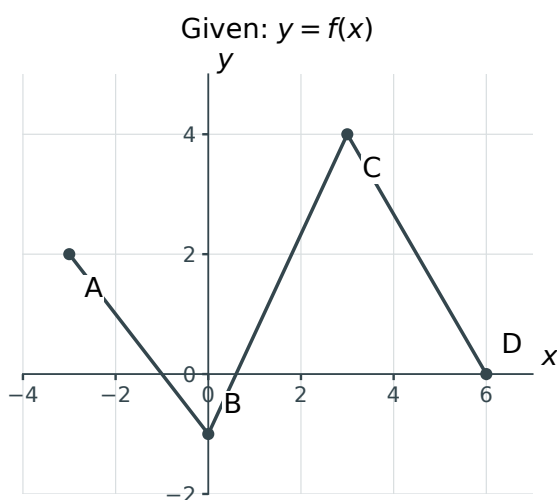
The domain of f is $[-3, 5)$ and its range is $(-2, 6]$. Let $g(x) = f(4 - 2x)$.

- List a valid sequence, with the horizontal shift last. [3]
- Find the domain of g . [2]
- State its range. [1]
- Find the image of $(1, 4)$. [1]

53. [Maximum mark: 11]

The graph joins $A(-3, 2)$, $B(0, -1)$, $C(3, 4)$ and $D(6, 0)$ by straight segments, including the endpoints. Let $g(x) = -2f(3x - 6) + 1$.

- Find the images of the four labelled points. [4]
- Sketch g on the supplied axes. [2]
- State its domain. [1]
- State its range. [1]
- List a valid ordered sequence from f to g . [3]



54. [Maximum mark: 10]

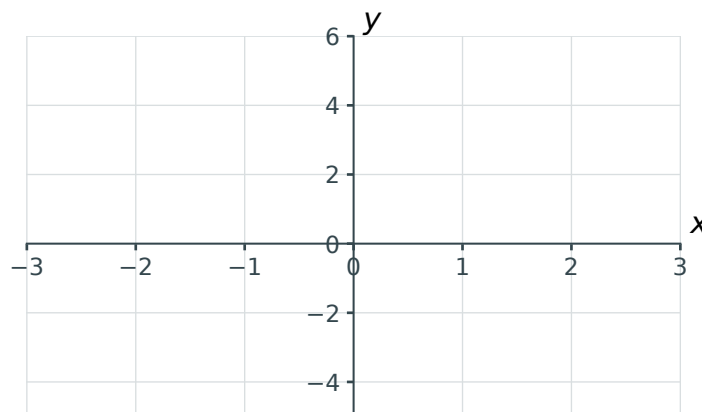
Under the transformation $g(x) = af(bx + c) + d$, the points $P = (1, 2)$ and $Q = (5, -1)$ map to $P' = (3, -3)$ and $Q' = (5, 6)$.

- (a) Find b and c . [3]
- (b) Find a and d . [3]
- (c) Factor the input expression. [1]
- (d) State a valid ordered transformation sequence. [3]

55. [Maximum mark: 12]

Let $g(x) = 4 - 2\sqrt{6 - 3x}$.

- (a) From $y = \sqrt{x}$, list a valid sequence with all horizontal changes first and each translation last within its category. [4]
- (b) State the domain of g . [1]
- (c) State its range. [1]
- (d) State its endpoint. [1]
- (e) Find both intercepts. [3]
- (f) Sketch g on the supplied axes. [2]


56. [Maximum mark: 13]

Let $f(x) = 1/x$ and $g(x) = -6f(2x + 4) + 1$.

- (a) Describe a valid sequence corresponding directly to the stated input and output operations. [4]
- (b) Simplify $g(x)$. [1]
- (c) Give an alternative sequence using no horizontal stretch. [2]
- (d) State its domain. [1]
- (e) State its range. [1]
- (f) State both asymptotes. [2]
- (g) Find both intercepts. [2]

57. [Maximum mark: 8]

Let S_p denote a horizontal stretch with SF $p > 0$, and T_h a translation h units to the right, where $h \neq 0$.

- (a) Find the final point when S_p is followed by T_h . [2]
- (b) Find the final point in the reverse order. [2]
- (c) Determine when the two point mappings are identical. [2]
- (d) Find the translation that, followed by S_p , produces the mapping in part (a). [2]

58. [Maximum mark: 10]

A function f has domain $[-4, -1) \cup (2, 8]$ and range $[-2, 5)$. Define $g(x) = -3f(2 - 2x) + 4$.

- (a) Find the domain of g , retaining the correct endpoints. [3]
- (b) Find the range of g . [2]
- (c) Find the images of $A = (-4, -2)$ and $B = (8, 1)$. [2]
- (d) State a valid ordered transformation sequence. [3]

59. [Maximum mark: 9]

Let $g(x) = 2f(3x - 6) - 5$.

- (a) Express $f(x)$ in terms of g . [3]
- (b) Describe a sequence taking the graph of g back to the graph of f . [3]
- (c) The point $P = (3, 4)$ lies on f . Find its image on g . [1]
- (d) Verify that your reverse transformation recovers P . [2]

60. [Maximum mark: 11]

The function f has domain $[-2, 4]$, range $[-1, 3]$, and contains $P = (2, 1)$. Define

$$g(x) = -2f(2x - 6) + 5, \quad h(x) = 3g(-x) + 1.$$

- (a) Express $h(x)$ directly in terms of f . [2]
- (b) Find the direct point mapping from f to h . [3]
- (c) State the domain of h . [1]
- (d) State its range. [1]
- (e) Find the final image of P . [1]
- (f) Give a valid sequence from f to h , completing all horizontal changes before the vertical changes. [3]

Answer key and solution outlines

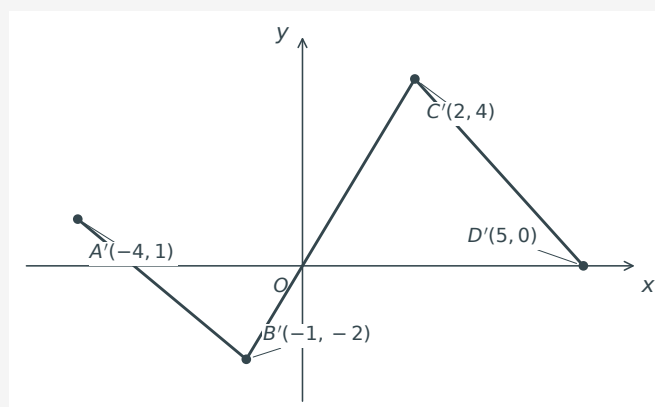
One valid sequence is given when several are possible. Equivalent sequences with correctly adjusted shifts are accepted. Sketches are indicative; labelled coordinates and asymptotes determine their positions.

1. Short response

1. (a) Vertical translation 6 units up. (b) Horizontal translation 4 units right. (c) Vertical stretch, SF $\frac{1}{3}$.
2. (a) Horizontal stretch, SF $\frac{1}{5}$. (b) Horizontal stretch, SF 4. (c) Reflection across the x -axis. (d) Reflection across the y -axis.
3. (a) $(-2, 4)$; (b) $(-7, 7)$; (c) $(-2, 14)$; (d) $(-1, 7)$. Horizontal changes alter x ; vertical changes alter y .
4. (a) $(x, y) \mapsto (x, -y)$. (b) $(x, y) \mapsto (-x, y)$. (c) $(x, y) \mapsto (3x, y)$.
5. (a) Recover input 8: $2k = 8$, so $k = 4$. (b) The horizontal scale factor is $1/k = 1/4$.
6. (a) -3 and 5 . (b) -3 and 5 . (c) An output of zero remains zero when multiplied by a non-zero constant.
7. (a) $[-5, 2)$. Inputs are unchanged. (b) $(3, 10]$. Add 4 to each original output.
8. (a) $[-2, 5)$. Add 3 to each original input. (b) $(-1, 6]$. Outputs are unchanged.
9. (a) $(-10, 4]$. (b) The excluded upper boundary 5 maps to -10 ; the included value -2 maps to 4. Multiplication by -2 reverses the order.
10. (a) $y = f(x + 2) - 5$. (b) $y = -3f(x)$.
11. (a) $A' = (-2, 0)$, $B' = (0, 4)$, $C' = (3, -2)$. (b) Heights are doubled: A is fixed, B moves up and C moves down. A vertical translation moves every point by the same vertical amount.
12. (a) $x = -1 + 4 = 3$. (b) $y = 2 - 3 = -1$.
13. At $x = 5$, the input to f is $5 - 2 = 3$. Hence $f(5 - 2) = f(3) = 8$: the original input and output are recovered. The point has moved 2 units right.
14. (a) Solve $3x = -6$, giving the point $(-2, 4)$. (b) The new input is the old input divided by 3; outputs are unchanged.
15. (a) The point $(r, 0)$ moves to $(r, 5)$, which is not on the x -axis. (b) The new roots satisfy $f(x) + 5 = 0$, that is, $f(x) = -5$. Such inputs need not exist.

2. Extended response

16. (a) The mapping $(x, y) \mapsto (x, 2 - y)$ gives $A'(-4, 1)$, $B'(-1, -2)$, $C'(2, 4)$, $D'(5, 0)$. (b) Join the image points in the original order, with closed endpoints. (c) $[-4, 5]$. (d) $[-2, 4]$. (e) Both transformations are vertical, so all input coordinates are unchanged.



17. (a) $(x - 2)^2 - 6(x - 2) + 5 = x^2 - 10x + 21$. (b) $f(x) = (x - 1)(x - 5) = (x - 3)^2 - 4$: vertex $(3, -4)$ and intercepts $(1, 0)$, $(5, 0)$. (c) Add 2 to each input: vertex $(5, -4)$ and intercepts $(3, 0)$, $(7, 0)$. (d) $\mathbb{R}$. (e) $[-4, \infty)$.

2. Extended response (continued)

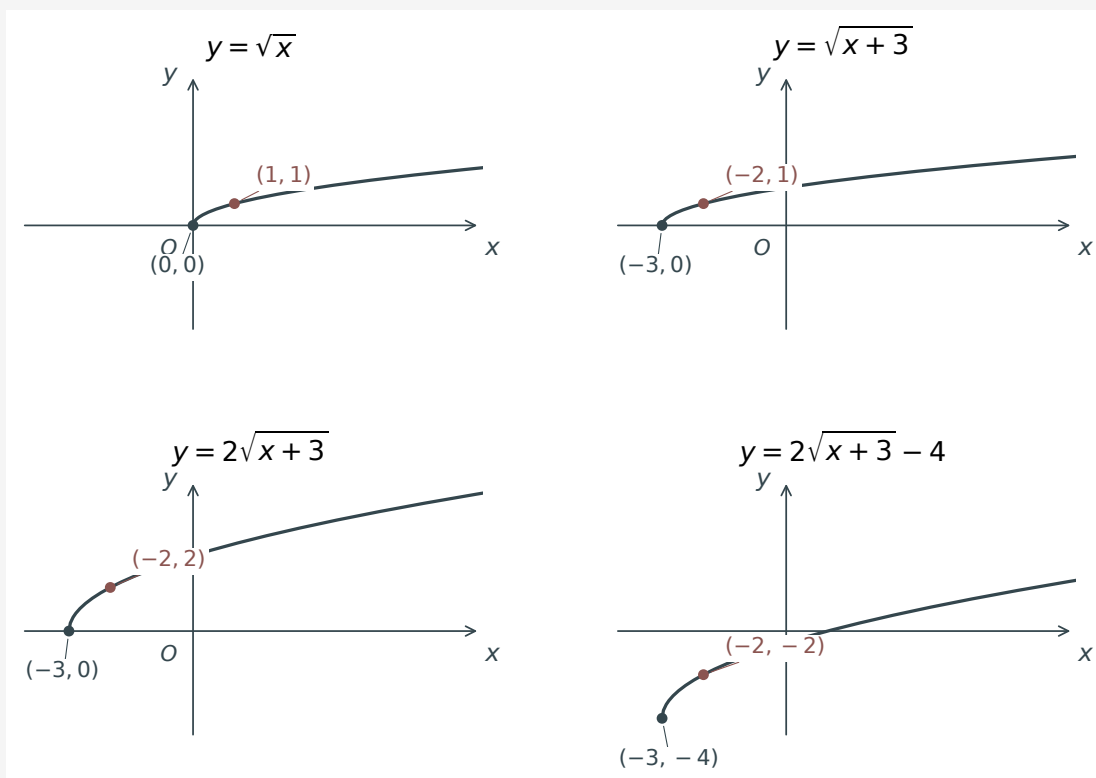
18. (a) Translate left 3, stretch vertically with SF 2, then translate down 4.

(b)–(e) The first row is the original graph; the following rows give the three stages.

Equation $y =$	Tracked point	Domain	Range
$\sqrt{x}$	(1, 1)	$[0, \infty)$	$[0, \infty)$
$\sqrt{x+3}$	(-2, 1)	$[-3, \infty)$	$[0, \infty)$
$2\sqrt{x+3}$	(-2, 2)	$[-3, \infty)$	$[0, \infty)$
$2\sqrt{x+3} - 4$	(-2, -2)	$[-3, \infty)$	$[-4, \infty)$

The endpoints are $(0, 0)$, $(-3, 0)$, $(-3, 0)$, $(-3, -4)$, all included.

(f) See the three transformed graphs below.



2. Extended response (continued)

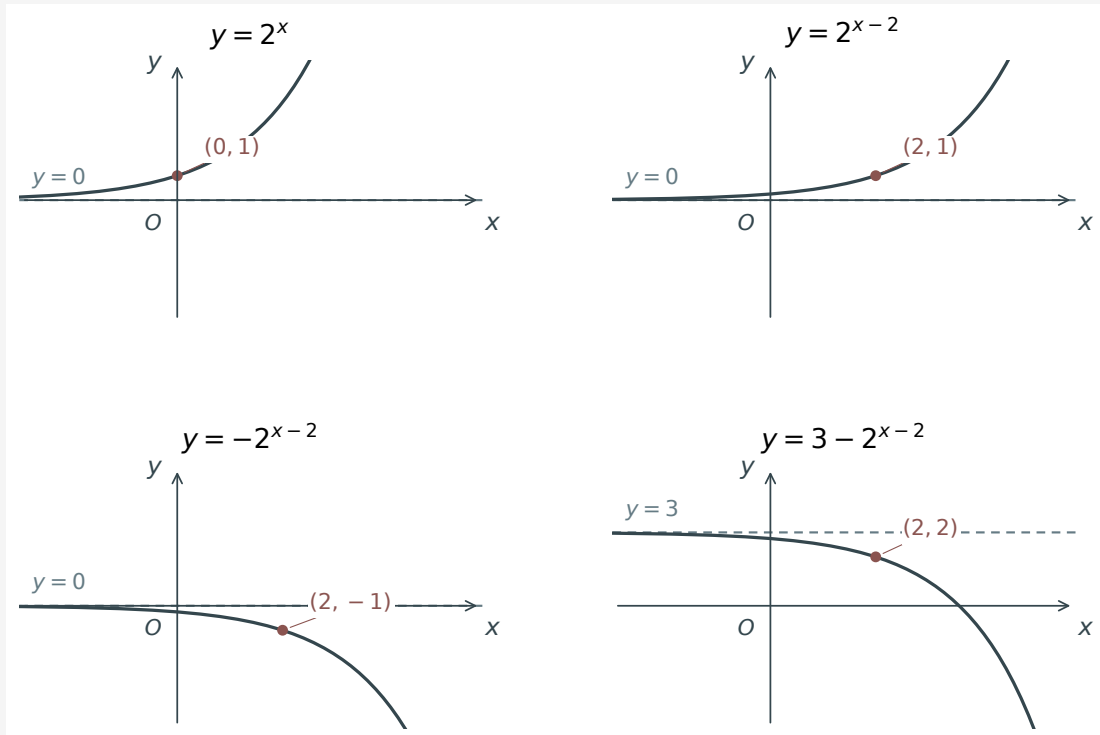
19. (a) Translate right 2, reflect across the x -axis, then translate up 3.

(b)–(e) The first row is the original graph; the following rows give the three stages.

Equation $y =$	Tracked point	Domain	Range
2^x	$(0, 1)$	$\mathbb{R}$	$(0, \infty)$
2^{x-2}	$(2, 1)$	$\mathbb{R}$	$(0, \infty)$
-2^{x-2}	$(2, -1)$	$\mathbb{R}$	$(-\infty, 0)$
$3 - 2^{x-2}$	$(2, 2)$	$\mathbb{R}$	$(-\infty, 3)$

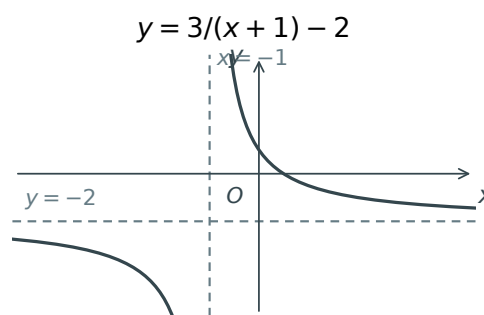
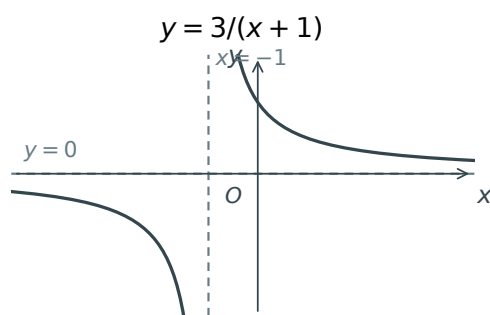
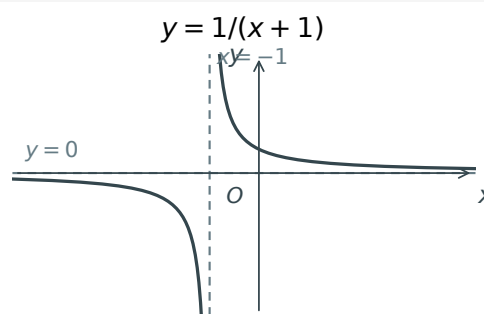
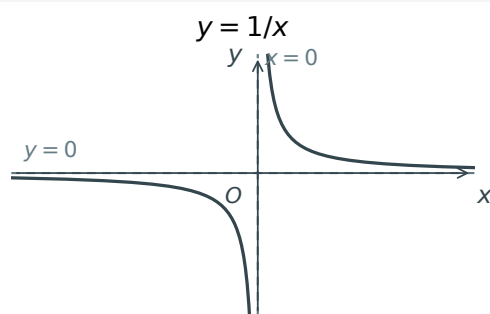
The horizontal asymptotes are $y = 0, 0, 0, 3$, respectively. No range boundary is attained.

(f) The three stage asymptotes are $y = 0, y = 0, y = 3$. (g) See the transformed graphs below.



2. Extended response (continued)

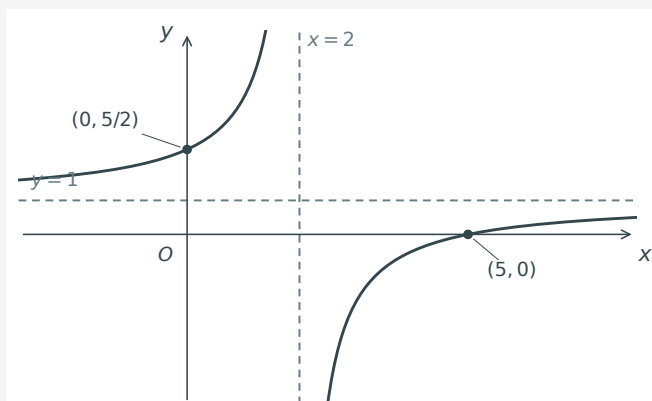
20. (a) $y = 1/(x+1)$, then $y = 3/(x+1)$, then $y = 3/(x+1) - 2$. **(b)** All three domains are $\mathbb{R} \setminus \{-1\}$. **(c)** The ranges are $\mathbb{R} \setminus \{0\}$, $\mathbb{R} \setminus \{0\}$ and $\mathbb{R} \setminus \{-2\}$. **(d)** Vertical asymptote $x = -1$ throughout; horizontal asymptotes $y = 0$, then -2 . **(e)** See the three transformed sketches below. **(f)** $[3 - 2(x+1)]/(x+1) = (1 - 2x)/(x+1)$, $x \neq -1$.



21. (a) Left 4; vertical stretch with scale factor 2; reflection across the x -axis; up 1. **(b)** $(-6, 2]$, subtracting 4 from the original inputs. **(c)** Outputs become $-2y + 1$: the excluded boundary 5 maps to -9 , and the included value -3 maps to 7. Range $(-9, 7]$. **(d)** $(2, -1) \mapsto (-2, 3)$.

22. (a) $4f(x) - 3$. **(b)** $4(f(x) - 3) = 4f(x) - 12$. **(c)** Use a downward shift of $3/4$, since $4(f(x) - 3/4) = 4f(x) - 3$.

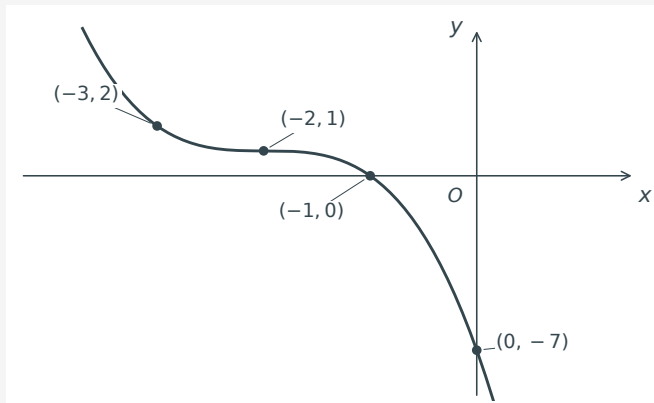
23. (a) Right 2; vertical stretch, scale factor 3; reflection across the x -axis; up 1. **(b)** $\mathbb{R} \setminus \{2\}$. **(c)** $\mathbb{R} \setminus \{1\}$. **(d)** $x = 2$ and $y = 1$. **(e)** $g(0) = 5/2$. Solving $3/(x-2) = 1$ gives $x = 5$. Intercepts $(0, 5/2)$ and $(5, 0)$. **(f)** See the sketch, with branches approaching both asymptotes.



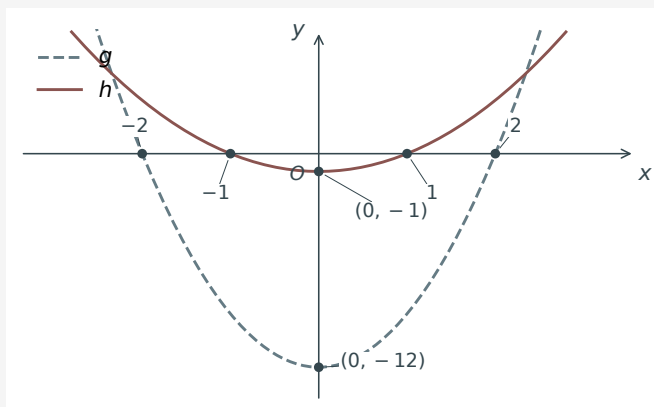
24. (a) $-3a + d = 10$ and $a + d = 2$, so $-4a = 8$, $a = -2$, $d = 4$. **(b)** Vertical stretch SF 2, reflection across the x -axis, then up 4. **(c)** Multiplication by a negative number reverses the order of the outputs.

2. Extended response (continued)

25. (a) Left 2, reflection across the x -axis, up 1. **(b)** $(x, y) \mapsto (x - 2, 1 - y)$ gives $(-3, 2), (-2, 1), (-1, 0)$. **(c)** $g(0) = -7$ and $g(x) = 0$ gives $x = -1$. Intercepts $(0, -7), (-1, 0)$. **(d)** $\mathbb{R}$. **(e)** $\mathbb{R}$. **(f)** See the decreasing cubic sketch. **(g)** Solve $-(x + 2)^3 + 1 = x$ numerically: $x \approx -0.786588$, hence $x = -0.787$ to 3 significant figures.



26. (a) $g(x) = 3x^2 - 12$; $h(x) = x^2 - 1$. **(b)** $(0, -12)$ for g ; $(0, -1)$ for h . **(c)** $g(x) = 0$ gives $x = \pm 2$; $h(x) = 0$ gives $x = \pm 1$. **(d)** See the two labelled parabolas. **(e)** The stretch preserves roots ± 2 , since multiplying a zero output by 3 leaves zero. The translation does not.



27. (a) The included lower output 2 becomes the maximum 7: $2a + d = 7$. Also $4a + d = 1$. Subtracting gives $2a = -6$, hence $a = -3$ and $d = 13$. **(b)** Vertical stretch, scale factor 3; reflection across the x -axis; translation 13 units up.

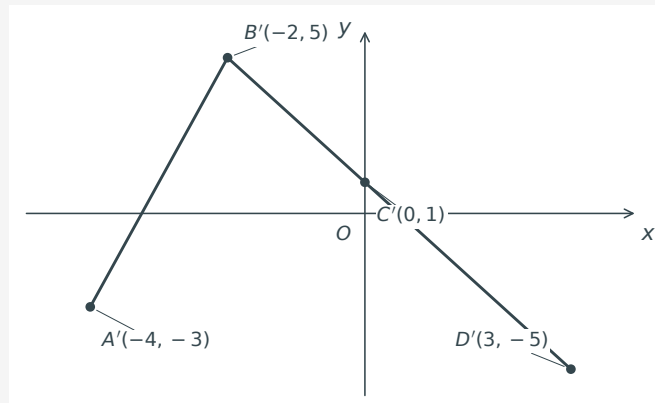
28. (a) $g: [-2, 4]$; $h: [-8, 16]$; $k: [-8, 4]$. **(b)** All three have range $[-2, 3]$: horizontal transformations preserve the outputs. **(c)** $g: (3, 1)$; $h: (12, 1)$; $k: (-6, 1)$.

29. (a) Horizontal stretch with scale factor 2, vertical stretch with scale factor $1/2$, then 1 unit up. **(b)** $0 \leq t \leq 24$ seconds. **(c)** $\frac{1}{2}(9) + 1 = 11/2$ metres. **(d)** $t/2 = 4$, so $t = 8$ seconds. **(e)** Solve $4.5e^{-0.08(t/2-4)^2} + 1 = 4$ on $[0, 24]$. The times are approximately 3.49742 s and 12.5026 s, hence 3.50 s and 12.5 s.

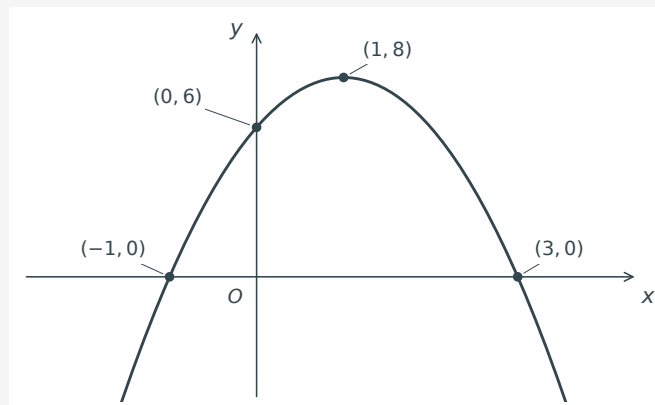
30. (a) Corresponding pairs: $(-1, -7), (1, 5), (4, -5), (8, 3)$. **(b)** $g(4) = -2f(3) + 3 = -8 + 3 = -5$. **(c)** Only four values of f are supplied; other inputs and outputs may exist.

3. IB exam-style questions

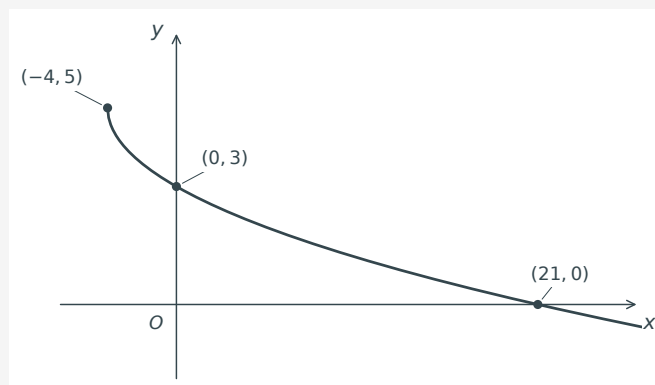
31. (a) Left 1, vertical stretch with scale factor 2, then down 1. **(b)** $(x, y) \mapsto (x - 1, 2y - 1)$ gives $A'(-4, -3)$, $B'(-2, 5)$, $C'(0, 1)$, $D'(3, -5)$. **(c)** Join the image points in their original order; endpoints included. **(d)** $[-4, 3]$. **(e)** $[-5, 5]$.



32. (a) Right 1, vertical stretch with scale factor 2, reflection across the x -axis, up 8. **(b)** $(1, 8)$. **(c)** $-2(x - 1)^2 + 8 = 0$ gives $(x - 1)^2 = 4$, hence $(-1, 0)$, $(3, 0)$. **(d)** $g(0) = 6$, giving $(0, 6)$. **(e)** $(-\infty, 8]$. **(f)** See the downward-opening parabola.



33. (a) Left 4, reflection across the x -axis, up 5. **(b)** $(-4, 5)$, included. **(c)** $[-4, \infty)$. **(d)** $(-\infty, 5]$. **(e)** $g(0) = 3$; $\sqrt{x + 4} = 5$ gives $x = 21$. Thus $(0, 3)$, $(21, 0)$. **(f)** See the decreasing square-root graph.



34. (a) The asymptote gives $d = 4$. Then $3a + 4 = -2$, so $a = -2$. **(b)** Vertical stretch with scale factor 2, reflection across the x -axis, up 4. **(c)** $\mathbb{R}$. **(d)** $(-\infty, 4)$. **(e)** $g(0) = 2$, giving $(0, 2)$.

35. (a) $h = 3, k = -2$. Since $a/(4 - 3) - 2 = 2$, $a = 4$. **(b)** Right 3, vertical stretch with scale factor 4, down 2. **(c)** $4/(x - 3) = 2$ gives $(5, 0)$. **(d)** $\mathbb{R} \setminus \{3\}$. **(e)** $\mathbb{R} \setminus \{-2\}$.

36. (a) The vertex is $(1, -6)$, so $h = 1, k = -6$. **(b)** The point $(3, 2)$ gives $2 = 4a - 6$, so $a = 2$. **(c)** Right 1, vertical stretch with scale factor 2, down 6. **(d)** $[-6, \infty)$. **(e)** $2(x - 1)^2 = 6$, so $x = 1 \pm \sqrt{3}$.

37. (a) $g(x) = 3f(x) + 8$. **(b)** Translate up $8/3$, since $3(f(x) + 8/3) = 3f(x) + 8$. **(c)** $(2, -1) \rightarrow (2, -3) \rightarrow (2, 5)$. **(d)** $(2, -1) \rightarrow (2, 5/3) \rightarrow (2, 5)$, giving the same final point.

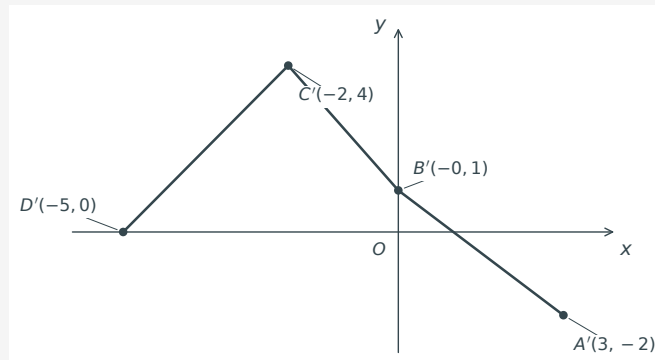
3. IB exam-style questions (continued)

38. (a) Translation 3 units left. **(b)** $2^{x+3} = 8 \cdot 2^x$, so the vertical scale factor is 8. **(c)** $(-3, 1)$. **(d)** $(0, 8)$. **(e)** Both points lie on g . The transformations associate each original point with different points on that same final graph.

39. (a) Right 3, vertical stretch with scale factor $1/2$, reflection across the x -axis, up 6. **(b)** $[2, 7]$: add 3 to the original inputs. **(c)** The included output 9 maps to $3/2$; the excluded boundary 2 maps to 5. The range is $[3/2, 5)$. **(d)** $(2, 6) \mapsto (5, 3)$.

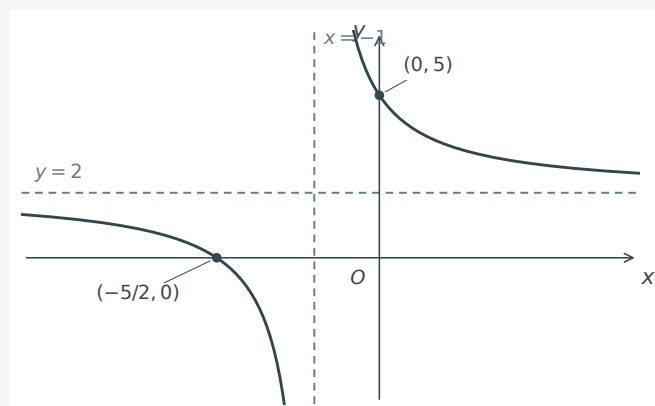
40. (a) $x + 3 = -4$ or 2 , so $x = -7$ or -1 . **(b)** $(-4, 5)$. **(c)** Roots of h require $f(x) = -3$, and these inputs are not given.

41. (a) Reflection across the y -axis. **(b)** $A'(3, -2)$, $B'(0, 1)$, $C'(-2, 4)$, $D'(-5, 0)$. **(c)** Join the image points in their original order, with endpoints included. **(d)** $[-5, 3]$. **(e)** $[-2, 4]$. **(f)** $x = 1$.



42. (a) $h = 2$, $k = -3$. **(b)** $5 = a\sqrt{6-2} - 3 = 2a - 3$, so $a = 4$. **(c)** Right 2, vertical stretch with scale factor 4, down 3. **(d)** $4\sqrt{x-2} = 3$ gives $x = 2 + 9/16 = 41/16$. **(e)** $[2, \infty)$. **(f)** $[-3, \infty)$.

43. (a) $2x + 5 = 2(x + 1) + 3$; divide by $x + 1$, with $x \neq -1$. **(b)** Left 1, vertical stretch with scale factor 3, up 2. **(c)** $\mathbb{R} \setminus \{-1\}$. **(d)** $\mathbb{R} \setminus \{2\}$. **(e)** $x = -1$, $y = 2$. **(f)** $g(0) = 5$ and $2x + 5 = 0$ gives $x = -5/2$. Intercepts $(0, 5)$, $(-5/2, 0)$. **(g)** See the two branches and dashed asymptotes.



44. (a) Horizontal stretch with scale factor 3, vertical stretch with scale factor $1/2$, up 4. **(b)** $[0, 24]$ hours. **(c)** $[9, 15]$ degrees Celsius, since the extreme outputs become $10/2 + 4 = 9$ and $22/2 + 4 = 15$. **(d)** Recover the original maximum input: $t/3 = 3$, so $t = 9$ hours.

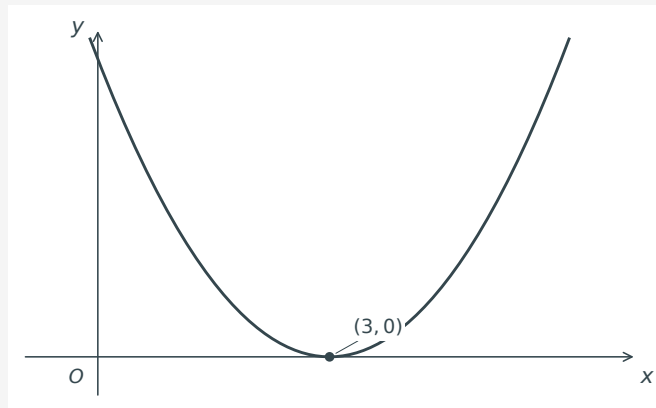
45. (a) $7 = 2f(-3) - 3$, so $f(-3) = 5$. **(b)** $-5 = 2f(2) - 3$, so $f(2) = -1$. **(c)** Undo $Y = 2y - 3$: $y = (Y + 3)/2$. The range is $[-2, 6]$. **(d)** Undo the right shift: subtract 4 from each input to get $[-4, 6]$.

46. (a) Reflect across the y -axis; stretch vertically with scale factor 3; reflect across the x -axis; translate up 2. **(b)** $[-7, 2]$. **(c)** $[-1, 14]$, since $1 \mapsto -1$ and $-4 \mapsto 14$. **(d)** $(3, 1) \mapsto (-3, -1)$. **(e)** A minimum: multiplication of outputs by a negative number reverses their order.

47. (a) $-f(-x) = -(x^2 + 2x - 3) = -x^2 - 2x + 3$. **(b)** The same equation, $-x^2 - 2x + 3$. **(c)** The point mappings are $(x, y) \rightarrow (x, -y) \rightarrow (-x, -y)$ and $(x, y) \rightarrow (-x, y) \rightarrow (-x, -y)$. The coordinates are changed independently, so the order does not matter.

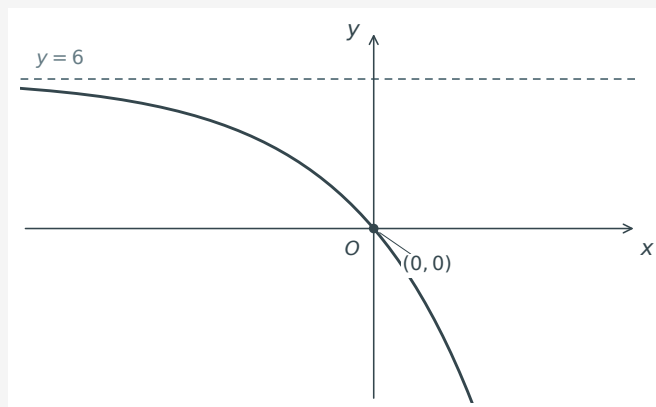
3. IB exam-style questions (continued)

48. (a) $g(x) = (x - 3)^2 - 4 + d$. (b) The minimum must be zero: $-4 + d = 0$, so $d = 4$. (c) $d > 4$. (d) $(3, 0)$. (e) $[0, \infty)$. (f) An upward-opening parabola touching the axis at $(3, 0)$.



49. (a) $y = pf(x) + d$. (b) $y = p(f(x) + d) = pf(x) + pd$. (c) Equality requires $d = pd$. Since $d > 0$, $p = 1$. (d) Both orders send the output y to $-py$.

50. (a) Left 1; vertical stretch with scale factor 3; reflection across the x -axis; up 6. (b) $\mathbb{R}$ at every stage. (c) For 2^{x+1} , $3 \cdot 2^{x+1}$, $-3 \cdot 2^{x+1}$, $6 - 3 \cdot 2^{x+1}$, the respective ranges are $(0, \infty)$, $(0, \infty)$, $(-\infty, 0)$, $(-\infty, 6)$. (d) $y = 0$, $y = 0$, $y = 0$, $y = 6$. (e) $g(0) = 0$ and $2^{x+1} = 2$ gives $x = 0$. Both intercepts coincide at $(0, 0)$. (f) See the decreasing exponential approaching $y = 6$ from below.



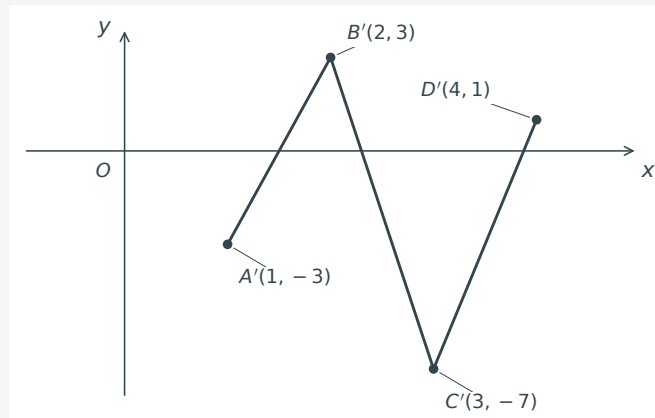
4. AA HL exclusive challenge

51. (a) $3x - 6 = 3(x - 2)$. (b) Horizontal stretch with scale factor $1/3$, then right 2. (c) Solve $3x - 6 = 9$: $x = 5$, giving $(5, -2)$. (d) Translate right 6, then stretch horizontally with scale factor $1/3$. Both mappings send x to $x/3 + 2$.

52. (a) $4 - 2x = -2(x - 2)$: horizontal stretch with scale factor $1/2$, reflection across the y -axis, then right 2. (b) $-3 \leq 4 - 2x < 5$ gives $-1/2 < x \leq 7/2$. Domain $(-1/2, 7/2]$. (c) $(-2, 6]$. (d) $4 - 2x = 1$ gives the point $(3/2, 4)$.

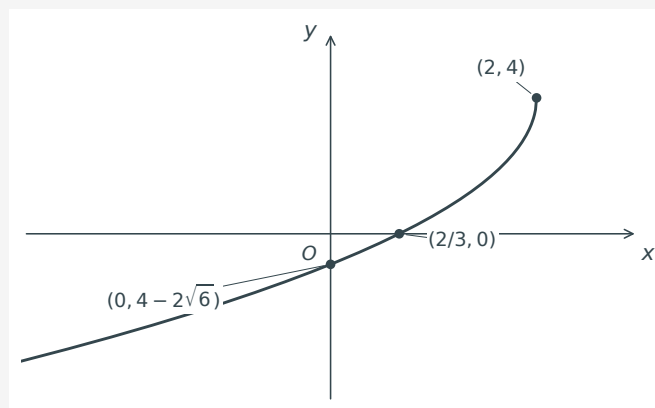
4. AA HL exclusive challenge (continued)

53. (a) Solve $3x' - 6 = x$: $x' = x/3 + 2$; also $y' = -2y + 1$. Hence $A'(1, -3)$, $B'(2, 3)$, $C'(3, -7)$, $D'(4, 1)$. **(b)** Join these points in the original order, with endpoints included. **(c)** $[1, 4]$. **(d)** $[-7, 3]$. **(e)** Horizontal stretch with scale factor $1/3$, right 2, vertical stretch with scale factor 2, reflection across the x -axis, up 1.



54. (a) $3b + c = 1$ and $5b + c = 5$, so $b = 2, c = -5$. **(b)** $2a + d = -3$ and $-a + d = 6$, so $a = -3, d = 3$. **(c)** $2x - 5 = 2(x - 5/2)$. **(d)** Horizontal stretch with scale factor $1/2$, right $5/2$, vertical stretch with scale factor 3, reflection across the x -axis, up 3.

55. (a) $6 - 3x = -3(x - 2)$: horizontal stretch with scale factor $1/3$, reflection across the y -axis, right 2; vertical stretch with scale factor 2, reflection across the x -axis, up 4. **(b)** $6 - 3x \geq 0$, giving $(-\infty, 2]$. **(c)** $(-\infty, 4]$. **(d)** $(2, 4)$, included. **(e)** $\sqrt{6 - 3x} = 2$ gives $x = 2/3$. The intercepts are $(2/3, 0)$ and $(0, 4 - 2\sqrt{6})$. **(f)** See the increasing curve ending at $(2, 4)$.



56. (a) Horizontal stretch with scale factor $1/2$, left 2, vertical stretch with scale factor 6, reflection across the x -axis, up 1. **(b)** $g(x) = -6/(2x + 4) + 1 = -3/(x + 2) + 1$. **(c)** Left 2, vertical stretch with scale factor 3, reflection across the x -axis, up 1. **(d)** $\mathbb{R} \setminus \{-2\}$. **(e)** $\mathbb{R} \setminus \{1\}$. **(f)** $x = -2, y = 1$. **(g)** $-3/(x + 2) + 1 = 0$ gives $(1, 0)$; $g(0) = -1/2$ gives $(0, -1/2)$.

57. (a) $(x, y) \mapsto (px, y) \mapsto (px + h, y)$. **(b)** $(x, y) \mapsto (x + h, y) \mapsto (px + ph, y)$. **(c)** Equality requires $h = ph$. Since $h \neq 0, p = 1$. **(d)** Translation by h/p in the positive x direction: $p(x + h/p) = px + h$. For negative h , this means left $|h|/p$.

58. (a) The input mapping is $x' = 1 - x/2$. The original intervals map to $(3/2, 3]$ and $[-3, 0)$, respectively. Thus the domain is $[-3, 0) \cup (3/2, 3]$. **(b)** $y' = -3y + 4$ gives $(-11, 10]$. **(c)** $A' = (3, 10)$ and $B' = (-3, 1)$. **(d)** Horizontal stretch with scale factor $1/2$, reflection across the y -axis, right 1; vertical stretch with scale factor 3, reflection across the x -axis, up 4.

59. (a) Set $t = x/3 + 2$ so $3t - 6 = x$. Then $f(x) = \frac{1}{2}g(x/3 + 2) + \frac{5}{2}$. **(b)** As $x/3 + 2 = (x + 6)/3$: horizontal stretch with scale factor 3, left 6, vertical stretch with scale factor $1/2$, then up $5/2$. **(c)** $P \mapsto (3, 3)$. **(d)** $(3, 3) \rightarrow (9, 3) \rightarrow (3, 3) \rightarrow (3, 3/2) \rightarrow (3, 4)$.

60. (a) $h(x) = 3[-2f(-2x - 6) + 5] + 1 = -6f(-2x - 6) + 16$. **(b)** Solve $-2x' - 6 = x$: $x' = -x/2 - 3$, while $y' = -6y + 16$. **(c)** $[-5, -2]$. **(d)** $[-2, 22]$. **(e)** $(-4, 10)$. **(f)** $-2x - 6 = -2(x + 3)$: horizontal stretch with scale factor $1/2$, reflection across the y -axis, left 3; vertical stretch with scale factor 6, reflection across the x -axis, up 16.