



Sequences, Series & Financial Mathematics – Practice 2

IB Mathematics: Analysis and Approaches SL & HL

Arithmetic and geometric sequences/series, financial applications, and infinite geometric series ■ 60 new questions

Learning goal	Build fluency and exam readiness with a second full practice set on arithmetic and geometric models, finite and infinite sums, sigma notation, compound interest, depreciation and real value.
Syllabus link	IB Mathematics AA SL/HL: SL 1.2 arithmetic sequences and series; SL 1.3 geometric sequences and series; SL 1.4 financial applications; SL 1.8 infinite convergent geometric sequences.
Structure	Section 1: 15 short-response questions. Section 2: 15 extended-response questions. Section 3: 20 IB-style exam questions. Section 4: 10 HL-exclusive IB-style challenge questions.
Technology	Questions marked with the calculator icon  are technology-appropriate / Paper 2-style. Unmarked questions in the IB-style sections are intended as non-calculator / Paper 1-style practice.

Instructions. Show clear mathematical working. Use exact values unless a decimal accuracy is requested. Identify the first term and common difference or ratio when setting up a sequence model. For an infinite geometric series, justify convergence using $|r| < 1$. Financial answers should include currency units and sensible rounding. Most questions are comparable in level to Practice 1; selected questions, particularly later in Sections 3 and 4, are deliberately a little more demanding.

Section 1: Short response

Questions 1–15

1. Consider the arithmetic sequence $31, 25, 19, 13, \dots$. State u_1 and the common difference d .
2. An arithmetic sequence has $u_1 = -4$ and $d = 7$. Find u_{18} .
3. In an arithmetic sequence, $u_6 = 23$ and $u_{14} = 55$. Find the common difference and the first term.
4. Find S_{15} for the arithmetic sequence $9, 13, 17, \dots$.
5. Evaluate $\sum_{r=1}^{10} (5r - 3)$.
6. Consider the geometric sequence $160, 80, 40, 20, \dots$. State u_1 and the common ratio r .
7. A geometric sequence has $u_1 = 3$ and $r = -2$. Find u_7 .
8. In a geometric sequence, $u_3 = 18$ and $u_6 = 486$. Find the common ratio.



9. Find S_7 for the geometric sequence $2, 6, 18, \dots$
10. Find the sum to infinity of $20 - 5 + 1.25 - \dots$
11. State which of the following geometric series converge: (i) $r = 1$; (ii) $r = -0.92$. Give a reason.
12. A principal of €3600 earns simple interest at 2.75% per year for 5 years. Find the final amount.
13. €6800 is invested at 3.6% interest compounded annually for 4 years. Find the value of the investment to the nearest cent.
14. A laptop is worth €2400 and depreciates by 22% each year. Find its value after 3 years, to the nearest cent.
15. An investment of €12 000 grows by 5.4% in one year while inflation is 2.6%. Find the real value of the investment, measured in today's euros, after one year.

Section 2: Extended response

Questions 16–30

16. An arithmetic sequence satisfies $u_4 = 11$ and $u_{15} = 55$. Find u_1 and d , and hence find S_{30} .
17. An arithmetic sequence has $u_1 = 46$ and $d = -2$. The sum of the first n terms is 280. Find all possible positive integer values of n .
18. An arithmetic sequence has first term 7 and common difference 4. Consider its first 300 terms. Find the sum of all these terms *except* those whose indices are multiples of 3; that is, omit

$$u_3 + u_6 + u_9 + \dots + u_{300}.$$

19. The arithmetic sequence

$$12, 17, 22, 27, \dots$$

is considered up to its 200th term. Find the sum of the terms that remain after all even-indexed terms

$$u_2 + u_4 + u_6 + \dots + u_{200}$$

are removed.

20. €12 000 is placed in an account paying simple interest at 1.8% per year. Write the balance after n complete years as an arithmetic model and find the least number of complete years for which the balance exceeds €14 500.



21. A geometric sequence satisfies $u_2 = 6$ and $u_5 = 162$. Find u_1 and r , and hence find S_7 .
-
22. A geometric sequence has $u_1 = 18$ and $r = 1.18$. Find the least value of n for which $S_n > 300$.
-
23. A ball is dropped from a height of 10 m. After each bounce it rises to 70% of its previous maximum height.
- (a) Find the total distance travelled immediately before the fifth bounce.
- (b) Find the total distance travelled if the bouncing continues indefinitely.
-
24. Two employees start on an annual salary of €45 000. Employee A receives a fixed increase of €1800 at the start of each new year. Employee B receives a 3.5% increase at the start of each new year. Let year 1 be the starting salary year. Determine the first year in which B's annual salary exceeds A's.
-
25. €9000 is invested at a nominal annual interest rate of 5.1%, compounded quarterly. Find the value of the investment after 6 years, to the nearest cent.
-
26. A machine worth €45 000 depreciates by 14% each year. Find the least number of complete years after which its value is below €18 000.
-
27. €30 000 is invested at 4.8% compound interest per year. Inflation is 2.4% per year. Find the real value of the investment after 10 years, to the nearest euro.
-
28. The first three terms of a sequence are
- $$u_1 = k + 1, \quad u_2 = 2k - 3, \quad u_3 = 5k - 9.$$
- (a) Find the value of k for which the three terms are arithmetic.
- (b) Find the values of k for which the three terms are geometric.
-
29. Consider the infinite series
- $$\sum_{n=0}^{\infty} 10 \left(\frac{2k+1}{5} \right)^n.$$
- Find the range of k for which the series converges. Given that its sum is 25, find k .
-
30. Use an infinite geometric series to express $0.1363636 \dots$, where the digits 36 repeat indefinitely, as a fraction in simplest form.

Section 3: IB exam-style

Questions 31–50 ■ original questions

31. The first and sixth terms of an arithmetic sequence are 50 and 25 respectively.
- (a) Find the fourteenth term. [4 marks]
- (b) The sum of the first n terms is zero. Find n . [2 marks]

32. The seventh term of an arithmetic sequence is 9. The sum of the fifth and twelfth terms is 30. Find the first term and the common difference. [5 marks]

33. The first three terms of a sequence are $k + 3$, $2k + 1$ and $4k - 5$.

(a) Given that the sequence is arithmetic, find k . [3 marks]

(b) Hence find an expression for S_n , the sum of the first n terms. [4 marks]

34. An arithmetic sequence has first term 4 and common difference 3. Let S be the sum of its first 250 terms, and let

$$M = u_5 + u_{10} + u_{15} + \cdots + u_{250}.$$

(a) Find S . [2 marks]

(b) Find M . [2 marks]

(c) Hence find the sum of the terms among the first 250 whose indices are *not* multiples of 5. [2 marks]

35. A geometric sequence has first term 128, fifth term 8 and positive common ratio.

(a) Find the common ratio and the second term. [3 marks]

(b) Find S_7 . [2 marks]

36. A geometric sequence satisfies $u_3 = 45$ and $u_6 = 1215$. Find the first term, the common ratio and S_6 . [6 marks]

37. An infinite geometric sequence has first term 24 and second term -8 .

(a) Find the sum to infinity. [2 marks]

(b) Find the sum of the first five terms. [3 marks]

38. Let $x = 0.3181818\dots$, where the digits 18 repeat indefinitely. Express x as a fraction in simplest form by using a geometric series. [5 marks]

39.  Two graduates each begin on an annual salary of €52 000. Carla's salary increases by €2100 each year. Diego's salary increases by 3.7% each year.

(a) Write expressions for their salaries in year n . [3 marks]

(b) Find the first year in which Diego's annual salary exceeds Carla's. [3 marks]

(c) For that year, state both salaries to the nearest euro. [2 marks]

40.  Two savings accounts each start with €6500. Account A earns simple interest at 6% per year. Account B earns 5.8% compound interest annually.

(a) Find the value of each account after 5 years. [4 marks]

(b) Hence determine which account contains more money and by how much. [2 marks]

41.  A scooter is purchased for €18 000 and depreciates by 17% each year.

- (a) Write an expression for its value after n complete years. [2 marks]
- (b) Find its value after 4 years, to the nearest euro. [2 marks]
- (c) Find the first complete year after which its value is below €8000. [3 marks]

42. An investment of €8000 grows by 15% per year. Inflation is 5% per year.

- (a) Write an exact expression for the real value after 3 years. [3 marks]
- (b) Give the real value as a single fraction of euros. [4 marks]

43. The sum of the first n terms of a geometric sequence is

$$S_n = \sum_{r=1}^n 7 \left(\frac{3}{5} \right)^{r-1}.$$

- (a) Write down the first three terms. [2 marks]
- (b) Find an expression for S_n . [2 marks]
- (c) Find S_∞ . [3 marks]

44. Consider the infinite series

$$\sum_{n=0}^{\infty} 6 \left(\frac{k+1}{4} \right)^n.$$

- (a) Find the range of k for which the series converges. [4 marks]
- (b) Given that the sum to infinity is 8, find k . [3 marks]

45.  A ball is dropped from a height of 12 m and rebounds to 65% of its previous maximum height.

- (a) Find the maximum height after the third bounce. [2 marks]
- (b) Find the total distance travelled immediately before the fifth bounce. [3 marks]
- (c) Find the total distance travelled if the motion continues indefinitely. [3 marks]

46. An auditorium has 22 seats in the first row. Each successive row contains 4 more seats than the previous row.

- (a) Find the number of seats in row 18. [2 marks]
- (b) Find the total number of seats in the first 18 rows. [3 marks]
- (c) Find the least number of complete rows needed for the total number of seats to exceed 1500. [3 marks]

47.  A wildlife reserve contains 1200 deer at the start of a study. The population is modelled as increasing by 15% each year.

- (a) Write an expression for the population after n years. [2 marks]
- (b) Find the modelled population after 8 years, to the nearest whole deer. [2 marks]
- (c) The population is recorded at the start and then after each of the next 8 years. Find the sum of these nine modelled recorded populations, to the nearest whole deer. [4 marks]



48. The sum of the first n terms of an arithmetic sequence is

$$S_n = n(3n + 5).$$

- (a) Show that $u_n = 6n + 2$. [4 marks]
- (b) Hence state u_1 and the common difference. [2 marks]
- (c) Find u_{20} . [1 marks]

49. A geometric sequence has first term 81, positive common ratio, and third term 9. The first two terms of this geometric sequence are also the first and seventh terms respectively of an arithmetic sequence.

- (a) Find the common ratio of the geometric sequence and its second term. [3 marks]
- (b) Find the common difference of the arithmetic sequence. [2 marks]
- (c) Find the value of n for which the n th arithmetic term is zero, and hence find the sum of the first n arithmetic terms. [4 marks]

50. An infinite geometric series has common ratio $\frac{2}{5}$ and sum to infinity 35.

- (a) Find the first term. [3 marks]
- (b) Find the seventh term. [2 marks]
- (c) Find S_6 . [4 marks]

Section 4: HL-exclusive IB-style challenge

Questions 51–60 ■ higher algebraic/modeling demand

51. A sequence of eight rectangular frames is formed. Frame 1 has area 30 cm^2 . From one frame to the next, both the width and height are multiplied by $\frac{4}{3}$.

- (a) Show that the area of frame n is $30 \left(\frac{16}{9}\right)^{n-1} \text{ cm}^2$. [2 marks]
- (b) Find the mean area of the eight frames, giving your answer in the form $p \left[\left(\frac{16}{9}\right)^8 - 1\right]$, where $p \in \mathbb{Q}^+$. [3 marks]
- (c) Find the median area of the eight frames in the form $q \left(\frac{16}{9}\right)^3$, where $q \in \mathbb{Q}^+$. [3 marks]

52. For an arithmetic sequence, the sum of the first n terms is

$$S_n = 2n^2 + 6n.$$

- (a) Find an expression for u_n . [3 marks]
- (b) Hence find u_1 and u_4 . [2 marks]

A geometric sequence $\{v_n\}$ satisfies $v_2 = u_1$ and $v_5 = u_4$.

- (c) Find the common ratio of $\{v_n\}$ exactly. [2 marks]
- (d) Find v_8 . [1 marks]

53. An arithmetic sequence and a geometric sequence both have first term $a > 0$ and fifth term 16.

- (a) Show that the third term of the arithmetic sequence is $\frac{a+16}{2}$. [2 marks]
- (b) Given that the geometric sequence has positive common ratio, show that its third term is $4\sqrt{a}$. [3 marks]
- (c) Show that the third term of the arithmetic sequence is never less than the third term of the geometric sequence. State when equality occurs. [5 marks]

54. A geometric sequence has first term 3 and common ratio 2. Let S_n be the sum of its first n terms, and define

$$T_n = S_1 + S_2 + \cdots + S_n.$$

- (a) Show that $S_n = 3(2^n - 1)$. [2 marks]

- (b) Hence show that

$$T_n = 3(2^{n+1} - n - 2).$$

[5 marks]

- (c) Find T_{10} . [2 marks]

55. The first three terms of a sequence are

$$u_1 = k - 1, \quad u_2 = 2k + 2, \quad u_3 = 8k + 8.$$

- (a) Show that these terms form a geometric sequence if and only if $(k + 1)(k - 3) = 0$. [4 marks]

- (b) For $k = 3$, find S_{2m} in terms of m . [3 marks]

- (c) For $k = -1$, find the sum to infinity of the resulting sequence and justify why it exists. [2 marks]

56.  Consider

$$S_n = \sum_{r=0}^{n-1} 12 \left(\frac{2c-1}{5} \right)^r.$$

- (a) Given that S_n converges as $n \rightarrow \infty$, find the range of possible values of c . [3 marks]

- (b) For $c = 2$, find the least value of n such that $|S_\infty - S_n| < 0.005$. [5 marks]

57.  A car is bought for €38 000. Its value decreases by 20% in each of the first two years. After the second year, its value decreases by 9% in each subsequent year.

- (a) Find the value of the car after the first two years. [2 marks]

- (b) Find its value 8 years after purchase, to the nearest euro. [3 marks]

- (c) Find the least number of complete years after which its value is less than 25% of the original purchase price. [3 marks]

58.  Two employees each start on an annual salary of €55 000. Employee A receives a fixed annual increase of €2100. Employee B receives an annual increase of 3.4% of the previous year's salary.

- (a) Find the first year in which B's annual salary exceeds A's. [4 marks]

- (b) Find B's total earnings over the first 15 years, to the nearest euro. [3 marks]

- (c) Compare this with A's total earnings over the same period. [3 marks]

59.  €18 000 is invested at a nominal annual interest rate of 4.6%, compounded monthly. Inflation is 2.3% per year.

- (a) Find the nominal value after 9 years, to the nearest cent. [4 marks]

- (b) Find the real value after 9 years, measured in today's euros, to the nearest euro. [3 marks]

- (c) Calculate the percentage increase in the real value over the 9 years. [2 marks]



60.  A family of tennis balls all rebound to a fixed fraction r of their previous maximum height, where $0 < r < 1$. A ball dropped from height H m travels a total distance δ m if it bounces indefinitely.

- (a) Show that $\delta = H \frac{1+r}{1-r}$. [3 marks]
- (b) Ball 1 is dropped from 12 m and travels 60 m in total. Find r . [2 marks]
- (c) The initial drop heights of Ball 1, Ball 2, Ball 3, ... form a geometric sequence with common ratio 0.8. Determine the first ball whose total distance is less than 20 m. [3 marks]
- (d) Find the sum of the total distances travelled by the first 10 balls. [2 marks]
- (e) For Ball 1, find the least bounce number m for which the maximum height after that bounce is less than 0.02 m. [2 marks]



Answer key and solution outlines

Questions 1–60

Section 1 answers: 1–15

1. $u_1 = 31$, $d = -6$.
2. $u_{18} = -4 + 17(7) = 115$.
3. $8d = 55 - 23 = 32$, so $d = 4$ and $u_1 = 23 - 5(4) = 3$.
4. $S_{15} = \frac{15}{2}[2(9) + 14(4)] = 555$.
5. $5 \sum_{r=1}^{10} r - 3(10) = 5(55) - 30 = 245$.
6. $u_1 = 160$, $r = \frac{1}{2}$.
7. $u_7 = 3(-2)^6 = 192$.
8. $r^3 = 486/18 = 27$, so $r = 3$.
9. $S_7 = 2 \frac{3^7-1}{3-1} = 2186$.
10. $r = -\frac{1}{4}$, so $S_\infty = \frac{20}{1+1/4} = 16$.
11. (i) diverges since $|1| \not< 1$; (ii) converges since $|-0.92| < 1$.
12. Interest = $3600(0.0275)(5) = \text{€}495$, so amount = $\text{€}4095$.
13. $6800(1.036)^4 = \text{€}7833.36$.
14. $2400(0.78)^3 = \text{€}1138.92$.
15. Real value = $12000 \frac{1.054}{1.026} \approx \text{€}12\,327.49$.

Section 2 answers: 16–30

16. $11d = 55 - 11 = 44 \Rightarrow d = 4$; $u_1 = 11 - 3(4) = -1$. Then $S_{30} = \frac{30}{2}[-2 + 29(4)] = 1710$.
17. $S_n = \frac{n}{2}[92 - 2(n-1)] = n(47-n)$. Hence $n(47-n) = 280$, so $(n-7)(n-40) = 0$ and $n = 7$ or 40 .
18. $u_n = 7 + 4(n-1) = 4n + 3$. Hence $S_{300} = \frac{300}{2}(7 + 1203) = 181500$. The omitted terms $u_3, u_6, \dots, u_{300}$ form an arithmetic sequence with 100 terms, first term 15 and last term 1203, so their sum is $\frac{100}{2}(15 + 1203) = 60900$. Required sum = $181500 - 60900 = \boxed{120600}$.
19. $u_{200} = 12 + 199(5) = 1007$, so $S_{200} = \frac{200}{2}(12 + 1007) = 101900$. The even-indexed terms $u_2, u_4, \dots, u_{200}$ form an arithmetic sequence with 100 terms, first term 17 and last term 1007, so their sum is $\frac{100}{2}(17 + 1007) = 51200$. Required sum = $101900 - 51200 = \boxed{50700}$.
20. Annual simple interest = $12000(0.018) = 216$, so $B_n = 12000 + 216n$. Solve $12000 + 216n > 14500$: $n > 11.57 \dots$, hence $n = 12$.
21. $r^3 = 162/6 = 27 \Rightarrow r = 3$; $u_1 = 2$. Hence $S_7 = 2 \frac{3^7-1}{3-1} = 2186$.
22. $S_n = 18 \frac{1.18^n-1}{0.18}$. $S_8 \approx 275.89 < 300$ and $S_9 \approx 343.55 > 300$, so $n = 9$.
23. (a) $10 + 2[7 + 4.9 + 3.43 + 2.401] = 45.462$ m. (b) $10 + 2 \left(\frac{7}{1-0.7} \right) = \frac{170}{3} \approx 56.67$ m.
24. $A_n = 45000 + 1800(n-1)$ and $B_n = 45000(1.035)^{n-1}$. The first crossover is year 10: $A_{10} = \text{€}61\,200$, $B_{10} \approx \text{€}61\,330.38$.
25. $9000 \left(1 + \frac{0.051}{4} \right)^{24} \approx \text{€}12\,198.22$.
26. $45000(0.86)^6 \approx \text{€}18\,205.53$ while $45000(0.86)^7 \approx \text{€}15\,656.75$, so 7 years.
27. Real value = $30000 \left(\frac{1.048}{1.024} \right)^{10} \approx \text{€}37\,821$.
28. Arithmetic: $2(2k-3) = (k+1) + (5k-9)$ gives $k = 1$. Geometric: $(2k-3)^2 = (k+1)(5k-9)$ gives $k^2 + 8k - 18 = 0$, hence $k = -4 \pm \sqrt{34}$.
29. Convergence requires $\left| \frac{2k+1}{5} \right| < 1$, so $-3 < k < 2$. Also $\frac{10}{1-(2k+1)/5} = 25$, giving $k = 1$.
30. $0.1363636 \dots = 0.1 + 0.036(1 + 0.01 + 0.01^2 + \dots) = \frac{1}{10} + \frac{0.036}{0.99} = \frac{3}{22}$.

Section 3 answers: 31–40

31. $d = (25 - 50)/5 = -5$. (a) $u_{14} = 50 + 13(-5) = -15$. (b) $S_n = \frac{n}{2}[100 - 5(n-1)] = \frac{n}{2}(105 - 5n)$, so $n = 21$.
32. $u_7 = u_1 + 6d = 9$. Also $u_5 + u_{12} = 2u_1 + 15d = 30$. Since $2u_1 + 12d = 18$, $3d = 12$, so $d = 4$ and $u_1 = -15$.
33. (a) $2(2k+1) = (k+3) + (4k-5)$ gives $k = 4$. Terms are 7, 9, 11, ... (b) $S_n = \frac{n}{2}[14 + 2(n-1)] = n(n+6)$.
34. (a) $u_{250} = 4 + 249(3) = 751$, so $S = \frac{250}{2}(4 + 751) = 94375$. (b) $u_5 = 16$, $u_{250} = 751$, and the selected terms have index step 5, so their common difference is $5d = 15$. There are 50 such terms; hence $M = \frac{50}{2}(16 + 751) = 19175$. (c) Required sum = $S - M = \boxed{75200}$.
35. $128r^4 = 8 \Rightarrow r^4 = \frac{1}{16}$. Since $r > 0$, $r = \frac{1}{2}$. Thus $u_2 = 64$ and $S_7 = 128 \frac{1-(1/2)^7}{1-1/2} = 254$.



36. $r^3 = 1215/45 = 27 \Rightarrow r = 3$. Then $u_1 = 45/9 = 5$ and $S_6 = 5 \frac{3^6-1}{3-1} = 1820$.
37. $r = -\frac{1}{3}$. (a) $S_\infty = 24/(1 + 1/3) = 18$. (b) $S_5 = 24 \frac{1-(-1/3)^5}{1+1/3} = \frac{488}{27}$.
38. $x = 0.3 + 0.018(1 + 0.01 + 0.01^2 + \dots) = \frac{3}{10} + \frac{0.018}{0.99} = \frac{3}{10} + \frac{1}{55} = \frac{7}{22}$.
39. $C_n = 52000 + 2100(n-1)$; $D_n = 52000(1.037)^{n-1}$. The first crossover is year 7. $C_7 = \text{€}64\,600$ and $D_7 \approx \text{€}64\,666$.
40. Account A: $6500[1 + 0.06(5)] = \text{€}8450$. Account B: $6500(1.058)^5 \approx \text{€}8616.71$. B contains about $\text{€}166.71$ more.

Section 3 answers: 41–50

41. (a) $V_n = 18000(0.83)^n$. (b) $V_4 \approx \text{€}8542.50$, so $\text{€}8542$ to the nearest euro. (c) $V_4 > 8000$ but $V_5 \approx \text{€}7090.27 < 8000$, so 5 years.
42. (a) $8000 \left(\frac{1.15}{1.05}\right)^3 = 8000 \left(\frac{23}{21}\right)^3$. (b) $\frac{97\,336\,000}{9261}$ euros.
43. (a) $7, \frac{21}{5}, \frac{63}{25}$. (b) $S_n = \frac{35}{2} \left[1 - \left(\frac{3}{5}\right)^n\right]$. (c) $S_\infty = \frac{35}{2}$.
44. (a) $\left|\frac{k+1}{4}\right| < 1 \Rightarrow -5 < k < 3$. (b) $6/[1 - (k+1)/4] = 8 \Rightarrow k = 0$.
45. (a) $12(0.65)^3 = 3.2955$ m. (b) $12 + 2[7.8 + 5.07 + 3.2955 + 2.142075] = 48.61515$ m. (c) $12 + 2\left(\frac{7.8}{1-0.65}\right) = \frac{396}{7} \approx 56.57$ m.
46. (a) $u_{18} = 22 + 17(4) = 90$. (b) $S_{18} = \frac{18}{2}(22 + 90) = 1008$. (c) $S_n = \frac{n}{2}[44 + 4(n-1)] = 2n(n+10)$. $S_{22} = 1408$ and $S_{23} = 1518$, so 23 rows.
47. (a) $P_n = 1200(1.15)^n$. (b) $P_8 \approx 3670.83$, so 3671 deer. (c) $1200 \frac{1.15^9-1}{0.15} \approx 20\,143$ deer.
48. (a) $u_n = S_n - S_{n-1} = 3n^2 + 5n - [3(n-1)^2 + 5(n-1)] = 6n + 2$. (b) $u_1 = 8$, $d = 6$. (c) $u_{20} = 122$.
49. (a) $81r^2 = 9$ and $r > 0$, so $r = \frac{1}{3}$ and $u_2 = 27$. (b) $27 = 81 + 6d$, so $d = -9$. (c) $0 = 81 - 9(n-1)$ gives $n = 10$. Then $S_{10} = \frac{10}{2}(81 + 0) = 405$.
50. (a) $35 = u_1/(1 - 2/5)$, so $u_1 = 21$. (b) $u_7 = 21(2/5)^6 = \frac{1344}{15625}$. (c) $S_6 = 21 \frac{1-(2/5)^6}{1-2/5} = \frac{108927}{3125}$.

Section 4 HL answers: 51–55

51. (a) Each length scales by $\frac{4}{3}$, so area scales by $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$: $A_n = 30\left(\frac{16}{9}\right)^{n-1}$. (b) Mean = $\frac{1}{8} \cdot 30 \frac{(16/9)^8-1}{16/9-1} = \frac{135}{28} \left[\left(\frac{16}{9}\right)^8 - 1\right]$. (c) Median = $\frac{A_4+A_5}{2} = 15\left(\frac{16}{9}\right)^3 \left(1 + \frac{16}{9}\right) = \frac{125}{3} \left(\frac{16}{9}\right)^3$.
52. (a) $u_n = S_n - S_{n-1} = 4n + 4$. (b) $u_1 = 8$, $u_4 = 20$. (c) $r^3 = v_5/v_2 = 20/8 = \frac{5}{2}$, so $r = \sqrt[3]{\frac{5}{2}}$. (d) $v_8 = v_5 r^3 = 20\left(\frac{5}{2}\right) = 50$.
53. (a) The third term of a five-term arithmetic progression is the average of the first and fifth: $\frac{a+16}{2}$. (b) If the geometric third term is g , then $g^2 = a \cdot 16$, so $g = 4\sqrt{a}$ since the ratio is positive. (c) $\frac{a+16}{2} - 4\sqrt{a} = \frac{(\sqrt{a}-4)^2}{2} \geq 0$. Equality occurs when $a = 16$.
54. (a) $S_n = 3 \frac{2^n-1}{2-1} = 3(2^n-1)$. (b) $T_n = 3 \sum_{k=1}^n (2^k-1) = 3[(2^{n+1}-2)-n] = 3(2^{n+1}-n-2)$. (c) $T_{10} = 3(2^{11}-12) = 6108$.
55. (a) Geometric condition $(2k+2)^2 = (k-1)(8k+8)$ gives $4(k+1)(3-k) = 0$, hence $(k+1)(k-3) = 0$. (b) For $k = 3$, the sequence is $2, 8, 32, \dots$ with $r = 4$, so $S_{2m} = 2 \frac{4^{2m}-1}{4-1} = \frac{2(16^m-1)}{3}$. (c) For $k = -1$, the sequence is $-2, 0, 0, \dots$ with $r = 0$; $|r| < 1$, so $S_\infty = -2$.

Section 4 HL answers: 56–60

56. (a) $\left|\frac{2c-1}{5}\right| < 1 \Rightarrow -2 < c < 3$. (b) For $c = 2$, $r = \frac{3}{5}$ and $|S_\infty - S_n| = \frac{12(3/5)^n}{1-3/5} = 30(0.6)^n$. Since the remainder is about 0.005078 for $n = 17$ and 0.003047 for $n = 18$, the least value is $n = 18$.
57. (a) $38000(0.8)^2 = \text{€}24\,320$. (b) $38000(0.8)^2(0.91)^6 \approx \text{€}13\,811$. (c) 25% of the original value is $\text{€}9500$. After 11 years the value is about $\text{€}10\,407.25$; after 12 years it is about $\text{€}9470.60$. Hence 12 years.
58. $A_n = 55000 + 2100(n-1)$ and $B_n = 55000(1.034)^{n-1}$. (a) First crossover: year 9, with $A_9 = \text{€}71\,800$ and $B_9 \approx \text{€}71\,866.58$. (b) $\sum_{n=1}^{15} B_n = 55000 \frac{1.034^{15}-1}{0.034} \approx \text{€}1\,053\,463$. (c) $\sum_{n=1}^{15} A_n = \frac{15}{2}[110000 + 14(2100)] = \text{€}1\,045\,500$, so B earns about $\text{€}7963$ more over the first 15 years.
59. (a) Nominal value = $18000(1 + 0.046/12)^{108} \approx \text{€}27\,209.88$. (b) Real value = $27209.88/(1.023)^9 \approx \text{€}22\,174$. (c) Percentage real increase $\approx \frac{22174.10-18000}{18000} \times 100\% = 23.19\%$.
60. (a) $\delta = H + 2(Hr + Hr^2 + \dots) = H + \frac{2Hr}{1-r} = H \frac{1+r}{1-r}$. (b) $60 = 12 \frac{1+r}{1-r}$ gives $r = \frac{2}{3}$. Thus $\delta = 5H$. (c) $H_n = 12(0.8)^{n-1}$, so $\delta_n = 60(0.8)^{n-1}$. The first value below 20 is $\delta_6 = 19.6608$, so Ball 6. (d) $\sum_{n=1}^{10} \delta_n = 60 \frac{1-0.8^{10}}{1-0.8} \approx 267.79$ m. (e) $12\left(\frac{2}{3}\right)^m < 0.02$ first occurs at $m = 16$.