



Sequences, Series & Financial Mathematics

IB Mathematics: Analysis and Approaches SL & HL

Arithmetic and geometric sequences/series, financial applications, and infinite geometric series ■ 60 questions

Learning goal	Use arithmetic and geometric models fluently, including finite and infinite sums, sigma notation, compound interest, depreciation and real-value calculations.
Syllabus link	IB Mathematics AA SL/HL: SL 1.2 arithmetic sequences and series; SL 1.3 geometric sequences and series; SL 1.4 financial applications; SL 1.8 infinite convergent geometric sequences.
Structure	Section 1: 15 short-response questions. Section 2: 15 extended-response questions. Section 3: 20 IB-style exam questions. Section 4: 10 HL-exclusive IB-style challenge questions.
Technology	Section 1 is mainly mental/fluently practice. Section 2 is mixed practice and technology may be used for finance, threshold and decimal questions. Sections 3 and 4 contain a mixture of Paper 1-style and Paper 2-style questions: the calculator icon  marks questions for which calculator technology is permitted; unmarked questions are designed in a non-calculator IB style.

Instructions. Show clear mathematical working. Use exact values unless a decimal accuracy is requested. When using a geometric model, identify the first term and common ratio clearly. For an infinite geometric series, justify convergence using $|r| < 1$. Financial answers should include currency units and sensible rounding. A question marked with the calculator icon  is a calculator/Paper 2-style question; unmarked exam-style questions are non-calculator/Paper 1-style. The syllabus content in 1.2, 1.3, 1.4 and 1.8 is common to AA SL and AA HL; Section 4 raises the algebraic and modelling demand to HL level rather than introducing a separate AHL syllabus statement.

Section 1: Short response

Questions 1–15

1. Consider the arithmetic sequence $18, 13, 8, 3, \dots$. State u_1 and the common difference d .
2. An arithmetic sequence has $u_1 = 7$ and $d = 4$. Find u_{25} .
3. In an arithmetic sequence, $u_4 = 20$ and $u_{10} = 50$. Find the common difference and the first term.
4. Find S_{20} for the arithmetic sequence $12, 15, 18, \dots$.
5. Evaluate $\sum_{r=1}^{12} (3r + 2)$.
6. Consider the geometric sequence $81, 27, 9, \dots$. State u_1 and the common ratio r .
7. A geometric sequence has $u_1 = 5$ and $r = 2$. Find u_8 .
8. In a geometric sequence, $u_2 = 12$ and $u_5 = 324$. Find the common ratio.



9. Find S_8 for the geometric sequence $3, 6, 12, \dots$
10. Find the sum to infinity of $12 + 6 + 3 + \dots$
11. State which of the following geometric series converge: (i) $r = -1.2$; (ii) $r = -\frac{3}{4}$. Give a reason.
12. A principal of €2400 earns simple interest at 3.5% per year for 6 years. Find the final amount.
13. €5000 is invested at 4% interest compounded annually for 3 years. Find the value of the investment.
14. A car is worth €28 000 and depreciates by 15% each year. Find its value after 2 years.
15. An investment of €10 000 grows by 6% in one year while inflation is 2%. Find the real value of the investment, measured in today's euros, after one year.

Section 2: Extended response

Questions 16–30

16. An arithmetic sequence satisfies $u_5 = 14$ and $u_{17} = 50$. Find u_1 and d , and hence find S_{25} .
17. An arithmetic sequence has $u_1 = 40$ and $d = -2$. The sum of the first n terms is 310. Find all possible positive integer values of n .
18. The first five measured values in a process are

$$52.4, \quad 57.1, \quad 61.6, \quad 66.5, \quad 71.0.$$

The data are modelled approximately by an arithmetic sequence. Use the average change from the first to the fifth value to estimate the common difference, then predict the twelfth value. State why the model is only approximate.

19. The terms $4r + k$, for $r = 1, 2, \dots$, form an arithmetic sequence. Given that

$$\sum_{r=1}^{10} (4r + k) = 310,$$

find k and then find the sum of the first 20 terms.

20. €8000 is placed in an account paying simple interest at 2.4% per year. Write the balance after n complete years as an arithmetic model and find the least number of complete years for which the balance exceeds €9500.
21. A geometric sequence satisfies $u_3 = 20$ and $u_6 = 160$. Find u_1 and r , and hence find S_8 .



22. A geometric sequence has $u_1 = 12$ and $r = 1.25$. Find the least value of n for which $S_n > 250$. Give your answer as a positive integer.

23. A ball is dropped from a height of 8 m. After each bounce it rises to 60% of its previous maximum height.

(a) Find the total distance travelled immediately before the fourth bounce.

(b) Find the total distance travelled if the bouncing continues indefinitely.

24. Two employees start on an annual salary of €40 000. Employee A receives a fixed increase of €1500 at the start of each new year. Employee B receives a 3.5% increase at the start of each new year. Let year 1 be the starting salary year. Determine the first year in which B's annual salary exceeds A's.

25. €7500 is invested at a nominal annual interest rate of 4.8%, compounded monthly. Find the value of the investment after 5 years, to the nearest cent.

26. A machine worth €32 000 depreciates by 18% each year. Find the least number of complete years after which its value is below €10 000.

27. €20 000 is invested at 5.5% compound interest per year. Inflation is 2.1% per year. Find the real value of the investment after 8 years, to the nearest euro.

28. The first three terms of a sequence are

$$u_1 = k - 4, \quad u_2 = 2k + 1, \quad u_3 = 5k - 2.$$

(a) Find the value of k for which the three terms are arithmetic.

(b) Find the values of k for which the three terms are geometric.

29. Consider the infinite series

$$\sum_{n=0}^{\infty} 6 \left(\frac{k-2}{3} \right)^n.$$

Find the range of k for which the series converges. Given that its sum is 9, find k .

30. Use an infinite geometric series to express $0.\overline{27} = 0.272727\dots$ as a fraction in simplest form.

Section 3: IB exam-style

Questions 31–50 ■ original questions

31. The first and fifth terms of an arithmetic sequence are 45 and 25 respectively.

(a) Find the thirteenth term.

[4 marks]

(b) The sum of the first n terms is zero. Find n .

[2 marks]



32. The seventh term of an arithmetic sequence is 8. The sum of the fifth and eleventh terms is 28. Find the first term and the common difference. [5 marks]

33. The first three terms of a sequence are $k + 2$, $3k - 1$ and $7k - 8$.

(a) Given that the sequence is arithmetic, find k . [3 marks]

(b) Hence find an expression for S_n , the sum of the first n terms. [4 marks]

34. The sum of the first n terms of an arithmetic sequence is

$$S_n = 3n^2 + 2n.$$

Find u_1 , the common difference, and u_{20} . [6 marks]

35. A geometric sequence has first term 96 and fourth term 12.

(a) Find the common ratio and the second term. [3 marks]

(b) Find S_6 . [2 marks]

36. A geometric sequence satisfies $u_2 = 18$ and $u_5 = 486$. Find the first term, the common ratio and S_5 . [6 marks]

37. An infinite geometric sequence has first term 15 and second term $-\frac{15}{4}$.

(a) Find the sum to infinity. [2 marks]

(b) Find the sum of the first four terms. [3 marks]

38. Let $x = 0.4181818\dots$, where the digits 18 repeat indefinitely. Express x as a fraction in simplest form by using a geometric series. [5 marks]

39.  Two graduates each begin on an annual salary of €30 000. Amira's salary increases by €1200 each year. Benoit's salary increases by 4% each year.

(a) Write expressions for their salaries in year n . [3 marks]

(b) Find each salary in year 4. [4 marks]

40.  Two accounts each start with €5000. Account A earns simple interest at 10% per year. Account B earns 10% compound interest annually.

(a) Find the value of each account after 3 years. [4 marks]

(b) Hence find how much more money is in Account B. [2 marks]

41.  A car is purchased for €20 000 and depreciates by 20% each year.

(a) Write an expression for its value after n complete years. [2 marks]

(b) Find its value after 3 years. [2 marks]

(c) Find the first complete year after which its value is below €9000. [3 marks]

42. An investment of €10 000 grows by 20% per year. Inflation is 10% per year.

- (a) Write an exact expression for the real value after 2 years. [3 marks]
- (b) Simplify your answer to a single fraction of euros. [4 marks]

43. The sum of the first n terms of a geometric sequence is

$$S_n = \sum_{r=1}^n 5 \left(\frac{2}{3} \right)^{r-1}.$$

- (a) Write down the first three terms. [2 marks]
- (b) Find an expression for S_n . [2 marks]
- (c) Find S_∞ . [3 marks]

44. Consider the infinite series

$$\sum_{n=0}^{\infty} 4 \left(\frac{2k-1}{3} \right)^n.$$

- (a) Find the range of k for which the series converges. [4 marks]
- (b) Given that the sum to infinity is 6, find k . [3 marks]

45.  A ball is dropped from a height of 6 m and rebounds to half of its previous maximum height.

- (a) Find the maximum height after the fourth bounce. [2 marks]
- (b) Find the total distance travelled immediately before the fourth bounce. [3 marks]
- (c) Find the total distance travelled if the motion continues indefinitely. [3 marks]

46. An auditorium has 18 seats in the first row. Each successive row contains 3 more seats than the previous row.

- (a) Find the number of seats in row 25. [2 marks]
- (b) Find the total number of seats in the first 25 rows. [3 marks]
- (c) Find the least number of complete rows needed for the total number of seats to exceed 1000. [3 marks]

47.  A culture contains 500 bacteria at time $t = 0$ hours. The population doubles every hour.

- (a) Write an expression for the population after n hours. [2 marks]
- (b) Find the population after 5 hours. [2 marks]
- (c) The population is recorded at $t = 0, 1, 2, 3, 4, 5$. Find the sum of these six recorded populations. [4 marks]

48. The sum of the first n terms of an arithmetic sequence is

$$S_n = n(4n + 3).$$

- (a) Show that $u_n = 8n - 1$. [4 marks]
- (b) Hence state u_1 and the common difference. [2 marks]
- (c) Find u_{15} . [1 marks]



49. A geometric sequence has first term 64, positive common ratio, and third term 16. The first two terms of this geometric sequence are also the first and ninth terms respectively of an arithmetic sequence.

- (a) Find the common ratio of the geometric sequence and its second term. [3 marks]
- (b) Find the common difference of the arithmetic sequence. [2 marks]
- (c) Find the value of n for which the n th arithmetic term is zero, and hence find the sum of the first n arithmetic terms. [4 marks]

50. An infinite geometric series has common ratio $-\frac{1}{3}$ and sum to infinity 18.

- (a) Find the first term. [3 marks]
- (b) Find the sixth term. [2 marks]
- (c) Find S_5 . [4 marks]

Section 4: HL-exclusive IB-style challenge

Questions 51–60 ■ AHL-level demand

51. A sequence of ten rectangular frames is formed. Frame 1 has area 24 cm^2 . From one frame to the next, both the width and height are multiplied by $\frac{3}{2}$.

- (a) Show that the area of frame n is $24 \left(\frac{9}{4}\right)^{n-1} \text{ cm}^2$. [2 marks]
- (b) Find the mean area of the ten frames, giving your answer in the form $p \left[\left(\frac{9}{4}\right)^{10} - 1\right]$, where $p \in \mathbb{Q}^+$. [3 marks]
- (c) Find the median area of the ten frames in the form $q \left(\frac{9}{4}\right)^4$, where $q \in \mathbb{Q}^+$. [3 marks]

52. For an arithmetic sequence, the sum of the first n terms is

$$S_n = n^2 + 7n.$$

- (a) Find an expression for u_n . [3 marks]
- (b) Hence find u_1 and u_5 . [2 marks]

A geometric sequence $\{v_n\}$ satisfies $v_2 = u_1$ and $v_5 = u_5$.

- (c) Find the common ratio of $\{v_n\}$ exactly. [2 marks]
- (d) Find v_8 . [1 marks]

53. An arithmetic sequence and a geometric sequence both have first term $a > 0$ and third term 1.

- (a) Show that the second term of the arithmetic sequence is $\frac{a+1}{2}$. [2 marks]
- (b) Given that the geometric sequence has positive common ratio, show that its second term is $\sqrt{a}$. [2 marks]
- (c) Show that the second term of the arithmetic sequence is never less than the second term of the geometric sequence. State when equality occurs. [5 marks]

54. A geometric sequence has first term 2 and common ratio 3. Let S_n be the sum of its first n terms, and define

$$T_n = S_1 + S_2 + \cdots + S_n.$$

- (a) Show that $S_n = 3^n - 1$. [2 marks]
- (b) Hence show that

$$T_n = \frac{3^{n+1} - 3}{2} - n.$$

- (c) Find T_8 . [2 marks]

55. The first three terms of a sequence are

$$u_1 = k - 2, \quad u_2 = k + 2, \quad u_3 = 9k - 18.$$

- (a) Show that these terms form a geometric sequence if and only if $(k - 1)(k - 4) = 0$. [4 marks]
- (b) For $k = 4$, find S_{2m} in terms of m . [2 marks]
- (c) For $k = 1$, find S_{2m} in terms of m . [2 marks]
- (d) State why neither geometric sequence has a sum to infinity. [1 marks]

56.  Consider

$$S_n = \sum_{r=0}^{n-1} 8 \left(\frac{c-1}{2} \right)^r.$$

- (a) Given that S_n converges as $n \rightarrow \infty$, find the range of possible values of c . [3 marks]
- (b) For $c = 2.5$, find the least value of n such that $|S_\infty - S_n| < 0.01$. [5 marks]

57.  A car is bought for €42 000. Its value decreases by 18% during the first year. After the first year, its value decreases by 12% in each subsequent year.

- (a) Find the value of the car after the first year. [2 marks]
- (b) Find its value 10 years after purchase, to the nearest euro. [3 marks]
- (c) Find the least number of complete years after which the value is less than 20% of the original purchase price. [3 marks]

58.  Two employees each start on an annual salary of €60 000. Employee A receives a fixed annual increase of €2200. Employee B receives an annual increase of 3.2% of the previous year's salary.

- (a) Find the first year in which B's annual salary exceeds A's. [4 marks]
- (b) Find B's total earnings over the first 12 years, to the nearest euro. [3 marks]
- (c) Compare this with A's total earnings over the same period. [3 marks]

59.  €15 000 is invested at a nominal annual interest rate of 4.2%, compounded monthly. Inflation is 2.0% per year.

- (a) Find the nominal value after 7 years, to the nearest cent. [4 marks]
- (b) Find the real value after 7 years, measured in today's euros, to the nearest euro. [3 marks]
- (c) Explain briefly why the real value is less than the nominal value. [2 marks]

60.  A family of tennis balls all rebound to a fixed fraction r of their previous maximum height, where $0 < r < 1$. A ball dropped from height H m travels a total distance δ m if it bounces indefinitely.

- (a) Show that $\delta = H \frac{1+r}{1-r}$. [3 marks]
- (b) Ball 1 is dropped from 9 m and travels 45 m in total. Find r . [2 marks]
- (c) The initial drop heights of Ball 1, Ball 2, Ball 3, ... form an arithmetic sequence, with Ball 6 dropped from 7.5 m. Determine the first ball whose total distance is less than 30 m. [3 marks]
- (d) Find the sum of the total distances travelled by the first 12 balls. [2 marks]
- (e) For Ball 1, find the least bounce number m for which the maximum height after that bounce is less than 0.05 m. [2 marks]



Answer key and solution outlines

Questions 1–60

Section 1 answers: 1–15

1. $u_1 = 18, d = -5$.
2. $u_{25} = 7 + 24(4) = 103$.
3. $u_{10} - u_4 = 6d = 30$, so $d = 5$ and $u_1 = 20 - 3(5) = 5$.
4. $S_{20} = \frac{20}{2}[2(12) + 19(3)] = 810$.
5. $3 \sum_{r=1}^{12} r + 2(12) = 3(78) + 24 = 258$.
6. $u_1 = 81, r = \frac{1}{3}$.
7. $u_8 = 5(2)^7 = 640$.
8. $u_5/u_2 = r^3 = 324/12 = 27$, so $r = 3$.
9. $S_8 = 3(2^8 - 1) = 765$.
10. $S_\infty = 12/(1 - \frac{1}{2}) = 24$.
11. (i) diverges since $|-1.2| > 1$; (ii) converges since $|- \frac{3}{4}| < 1$.
12. Interest = $2400(0.035)(6) = \text{€}504$, so amount = $\text{€}2904$.
13. $5000(1.04)^3 = \text{€}5624.32$.
14. $28000(0.85)^2 = \text{€}20\,230$.
15. Real value = $10000 \frac{1.06}{1.02} = \text{€}10\,392.16$ (to cents).

Section 2 answers: 16–30

16. $12d = 36 \Rightarrow d = 3; u_1 = 14 - 4(3) = 2$. Then $S_{25} = \frac{25}{2}[4 + 24(3)] = 950$.
17. $310 = \frac{n}{2}[80 - 2(n-1)] = n(41-n)$. Hence $n^2 - 41n + 310 = 0 = (n-10)(n-31)$, so $n = 10$ or 31 .
18. Average change $d \approx (71.0 - 52.4)/4 = 4.65$. Prediction: $u_{12} \approx 52.4 + 11(4.65) = 103.55$. The observed consecutive differences are not identical.
19. $4(55) + 10k = 310 \Rightarrow k = 9$. Then $S_{20} = 4(210) + 20(9) = 1020$.
20. Annual simple interest = $8000(0.024) = 192$, so $B_n = 8000 + 192n$. Solve $8000 + 192n > 9500$: $n > 7.8125$, hence $n = 8$.
21. $r^3 = 160/20 = 8 \Rightarrow r = 2; u_1 = 20/4 = 5$. $S_8 = 5(2^8 - 1) = 1275$.
22. $S_n = 12 \frac{1.25^n - 1}{0.25} = 48(1.25^n - 1)$. Need $1.25^n > 6.20833 \dots$; $1.25^8 < 6.20833$ and $1.25^9 > 6.20833$, so $n = 9$.
23. Rebound heights: 4.8, 2.88, 1.728, ... (a) $8 + 2(4.8 + 2.88 + 1.728) = 26.816$ m. (b) $8 + 2 \left(\frac{4.8}{1-0.6} \right) = 32$ m.
24. $A_n = 40000 + 1500(n-1)$ and $B_n = 40000(1.035)^{n-1}$. Checking integer years gives the first crossover at $n = 6$: $A_6 = \text{€}47\,500$, $B_6 \approx \text{€}47\,507.45$.
25. $7500 \left(1 + \frac{0.048}{12} \right)^{60} = \text{€}9529.81$.
26. $32000(0.82)^n < 10000$. This first occurs at $n = 6$.
27. Real value = $20000 \left(\frac{1.055}{1.021} \right)^8 \approx \text{€}25\,992$.
28. Arithmetic: $2(2k+1) = (k-4) + (5k-2)$ gives $k = 4$. Geometric: $(2k+1)^2 = (k-4)(5k-2)$ gives $k^2 - 26k + 7 = 0$, hence $k = 13 \pm 9\sqrt{2}$.
29. Convergence requires $|\frac{k-2}{3}| < 1$, so $-1 < k < 5$. Also $\frac{6}{1-(k-2)/3} = 9$, giving $k = 3$.
30. $0.272727 \dots = 0.27(1 + 0.01 + 0.01^2 + \dots) = \frac{0.27}{0.99} = \frac{3}{11}$.

Section 3 answers: 31–40

31. $d = (25 - 45)/4 = -5$. (a) $u_{13} = 45 + 12(-5) = -15$. (b) $S_n = \frac{n}{2}[90 - 5(n-1)] = \frac{n}{2}(95 - 5n)$, so $n = 19$.
32. $u_7 = u_1 + 6d = 8$. Also $u_5 + u_{11} = 2u_1 + 14d = 28$, so $u_1 + 7d = 14$. Subtracting gives $d = 6$ and $u_1 = -28$.
33. (a) $2(3k-1) = (k+2) + (7k-8)$ gives $k = 2$. Terms are 4, 5, 6, ..., so $u_1 = 4, d = 1$. (b) $S_n = \frac{n}{2}[8 + (n-1)] = \frac{n(n+7)}{2}$.
34. $u_n = S_n - S_{n-1} = 6n - 1$. Thus $u_1 = 5, d = 6, u_{20} = 119$.
35. $96r^3 = 12 \Rightarrow r = \frac{1}{2}$. Hence $u_2 = 48$ and $S_6 = 96 \frac{1-(1/2)^6}{1-1/2} = 189$.



36. $r^3 = 486/18 = 27 \Rightarrow r = 3$. Then $u_1 = 6$ and $S_5 = 6 \frac{3^5-1}{3-1} = 726$.
37. $r = -\frac{1}{4}$. (a) $S_\infty = 15/(1 + 1/4) = 12$. (b) $S_4 = 15 \frac{1-(1/4)^4}{1+1/4} = \frac{765}{64}$.
38. $x = 0.4 + 0.018(1 + 0.01 + \dots) = \frac{2}{5} + \frac{0.018}{0.99} = \frac{2}{5} + \frac{1}{55} = \frac{23}{55}$.
39. (a) $A_n = 30000 + 1200(n-1)$; $B_n = 30000(1.04)^{n-1}$. (b) $A_4 = \text{€}33\,600$; $B_4 = 30000(1.04)^3 = \text{€}33\,745.92$.
40. (a) A: $5000 + 3(500) = \text{€}6500$. B: $5000(1.1)^3 = \text{€}6655$. (b) Difference = $\text{€}155$.

Section 3 answers: 41–50

41. (a) $V_n = 20000(0.8)^n$. (b) $V_3 = \text{€}10\,240$. (c) $V_3 > 9000$ but $V_4 = \text{€}8192 < 9000$, so 4 years.
42. (a) $10000 \left(\frac{1.2}{1.1}\right)^2$. (b) $10000 \left(\frac{1.2}{1.1}\right)^2 = \frac{1\,440\,000}{121}$ euros.
43. (a) $5, \frac{10}{3}, \frac{20}{9}$. (b) $S_n = 15 \left[1 - \left(\frac{2}{3}\right)^n\right]$. (c) $S_\infty = 15$.
44. (a) $\left|\frac{2k-1}{3}\right| < 1 \Rightarrow -1 < k < 2$. (b) $4/[1 - (2k-1)/3] = 6 \Rightarrow k = 1$.
45. (a) $6(1/2)^4 = \frac{3}{8}$ m. (b) $6 + 2(3 + 1.5 + 0.75) = \frac{33}{2}$ m. (c) $6 + 2\left(\frac{3}{1-1/2}\right) = 18$ m.
46. (a) $u_{25} = 18 + 24(3) = 90$. (b) $S_{25} = \frac{25}{2}(18 + 90) = 1350$. (c) $S_n = \frac{n}{2}[36 + 3(n-1)] = \frac{3n(n+11)}{2}$. $S_{20} = 930$ and $S_{21} = 1008$, so 21 rows.
47. (a) $P_n = 500(2^n)$. (b) $P_5 = 16\,000$. (c) $500(1 + 2 + \dots + 2^5) = 500(63) = 31\,500$.
48. (a) $u_n = S_n - S_{n-1} = 4n^2 + 3n - [4(n-1)^2 + 3(n-1)] = 8n - 1$. (b) $u_1 = 7$, $d = 8$. (c) $u_{15} = 119$.
49. (a) $64r^2 = 16$ and $r > 0$, so $r = \frac{1}{2}$ and second term 32. (b) $32 = 64 + 8d$, so $d = -4$. (c) $0 = 64 + (n-1)(-4)$ gives $n = 17$. Then $S_{17} = \frac{17}{2}(64 + 0) = 544$.
50. (a) $18 = u_1/(1 + 1/3)$, so $u_1 = 24$. (b) $u_6 = 24(-1/3)^5 = -\frac{8}{81}$. (c) $S_5 = 24 \frac{1-(-1/3)^5}{1+1/3} = \frac{488}{27}$.

Section 4 HL answers: 51–55

51. (a) Each length scales by $\frac{3}{2}$, so area scales by $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. $A_n = 24\left(\frac{9}{4}\right)^{n-1}$. (b) Mean = $\frac{1}{10} \cdot 24 \frac{(9/4)^{10}-1}{9/4-1} = \frac{48}{25} \left[\left(\frac{9}{4}\right)^{10} - 1\right]$. (c) Median = $\frac{A_5+A_6}{2} = 12\left(\frac{9}{4}\right)^4 \left(1 + \frac{9}{4}\right) = 39\left(\frac{9}{4}\right)^4$.
52. (a) $u_n = S_n - S_{n-1} = 2n + 6$. (b) $u_1 = 8$, $u_5 = 16$. (c) $v_5/v_2 = r^3 = 2$, so $r = \sqrt[3]{2}$. (d) $v_8 = v_5 r^3 = 16(2) = 32$.
53. (a) Arithmetic middle term is the average: $\frac{a+1}{2}$. (b) For positive ratio, $s^2 = a \cdot 1$, so $s = \sqrt{a}$. (c) $\frac{a+1}{2} - \sqrt{a} = \frac{(\sqrt{a}-1)^2}{2} \geq 0$. Equality occurs only when $a = 1$.
54. (a) $S_n = 2 \frac{3^n-1}{3-1} = 3^n - 1$. (b) $T_n = \sum_{k=1}^n (3^k - 1) = \frac{3(3^n-1)}{2} - n = \frac{3^{n+1}-3}{2} - n$. (c) $T_8 = 9832$.
55. (a) Geometric condition $(k+2)^2 = (k-2)(9k-18)$ gives $8(k-1)(k-4) = 0$. (b) $k = 4$ gives $2, 6, 18, \dots$, so $S_{2m} = 2 \frac{3^{2m}-1}{3-1} = 9^m - 1$. (c) $k = 1$ gives $-1, 3, -9, \dots$, so $S_{2m} = -\frac{1-(-3)^{2m}}{1+3} = \frac{9^m-1}{4}$. (d) In both cases $|r| = 3 > 1$.

Section 4 HL answers: 56–60

56. (a) $\left|\frac{c-1}{2}\right| < 1 \Rightarrow -1 < c < 3$. (b) For $c = 2.5$, $r = 0.75$ and the remainder is $|S_\infty - S_n| = \frac{8(0.75)^n}{1-0.75} = 32(0.75)^n$. Solve $32(0.75)^n < 0.01$; $n > 28.0549 \dots$, so $n = 29$.
57. (a) $42000(0.82) = \text{€}34\,440$. (b) $42000(0.82)(0.88)^9 \approx \text{€}10\,900$. (c) Solve $42000(0.82)(0.88)^{n-1} < 8400$; the first integer solution is $n = 13$.
58. $A_n = 60000 + 2200(n-1)$, $B_n = 60000(1.032)^{n-1}$. (a) First crossover: year 11 ($A_{11} = \text{€}82\,000$, $B_{11} \approx \text{€}82\,214.46$). (b) $\sum_{n=1}^{12} B_n = 60000 \frac{1.032^{12}-1}{0.032} \approx \text{€}861\,262$. (c) $\sum_{n=1}^{12} A_n = \frac{12}{2}[120000 + 11(2200)] = \text{€}865\,200$, so A earns about $\text{€}3938$ more over the first 12 years even though B has the larger annual salary by year 11.
59. (a) Nominal value = $15000(1 + 0.042/12)^{84} \approx \text{€}20\,116.43$. (b) Real value = $20116.43/(1.02)^7 \approx \text{€}17\,513$. (c) Inflation reduces purchasing power, so the same nominal amount is worth fewer today's euros.
60. (a) $\delta = H + 2(Hr + Hr^2 + \dots) = H + \frac{2Hr}{1-r} = H \frac{1+r}{1-r}$. (b) $45 = 9 \frac{1+r}{1-r}$ gives $r = \frac{2}{3}$. (c) Initial heights are arithmetic with $H_1 = 9$, $H_6 = 7.5$, so $d = -0.3$ and $H_n = 9 - 0.3(n-1)$. Since $\delta_n = 5H_n$, require $5H_n < 30 \Rightarrow H_n < 6$, first at $n = 12$. (d) $\delta_1 = 45$, $\delta_{12} = 28.5$, so sum = $\frac{12}{2}(45 + 28.5) = 441$ m. (e) $9\left(\frac{2}{3}\right)^m < 0.05$ gives $m > 12.807 \dots$, so $m = 13$.