COURSE REVISION GUIDE

Analysis and approaches

For students taking IB Mathematics - Higher level

83 numbered sections | 5 topics | AA HL

EXAMINATIONS 2026-2028

Current course: first assessment 2021; final assessment November 2028. Checked against the official guide updated November 2024.

Connect ideas, justify your arguments and apply familiar methods to unfamiliar problems.

Use each checkbox after you can carry out the skill and explain your method. Return to unchecked skills, practice an example, then try a fresh question without the solution beside you.

YOUR COURSE MAP

1	**Number and algebra**	16 sections	**p. 2**
2	**Functions**	16 sections	**p. 4**
3	**Geometry and trigonometry**	18 sections	**p. 6**
4	**Statistics and probability**	14 sections	**p. 9**
5	**Calculus**	19 sections	**p. 11**

Skills beyond the checklist

Clear mathematical habits

Show your method; check restrictions, units and accuracy; justify what your answer means.

Your mathematical exploration

Investigate a focused question, develop your own mathematics, evaluate results and acknowledge sources.

Official syllabus and course versions

Open the official IB-hosted subject guide

2029 examinations onward: check the revised course information

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1 Number and algebra

Build reliable algebraic tools, recognise numerical patterns and justify why a result works.

1.1 Working across scales

- ☐ Calculate with very large or very small quantities and express the result in standard form.
- ☐ Keep the leading number from 1 up to, but not including, 10; translate a calculator's E display into a power of 10.

1.2 Patterns with a constant step

- ☐ Recover the first term and common difference from information about an arithmetic sequence.
- ☐ Find a particular term or a total of consecutive terms, including sums written with Σ.
- ☐ Build an arithmetic model for a context, make predictions and judge whether a constant step is a reasonable approximation.

1.3 Patterns with a constant multiplier

- ☐ Identify the first term and common ratio of a geometric sequence, including from incomplete information.
- ☐ Calculate terms and finite totals, and move between an expanded sum and Σ notation.
- ☐ Use a geometric model to describe repeated percentage change and interpret predictions in context.

1.4 Money over time

- ☐ Model compound growth and annual depreciation; distinguish a percentage change from the multiplier used in the model.
- ☐ Match the interest rate and number of periods to annual, half-yearly, quarterly or monthly compounding; use financial technology when appropriate.
- ☐ Adjust an investment's future value for inflation to describe its purchasing power.

1.5 Powers and logarithmic notation

- ☐ Simplify products, quotients and powers involving integer exponents, including zero and negative exponents.
- ☐ Translate an exponential statement into a logarithmic statement and explain which exponent a logarithm represents.
- ☐ Evaluate common and natural logarithms with technology, remembering that a real logarithm needs a positive argument.

1.6 Writing a convincing algebraic argument

- ☐ Establish a numerical or algebraic claim through a logical chain of justified steps.
- ☐ Prove an identity by transforming one side into the other; distinguish an identity from an equation true only for particular values.
- ☐ Check a proposed result and explain what the check does, and does not, establish.

1.7 Rewriting powers and solving exponential equations

- ☐ Connect rational powers with roots and simplify expressions, taking the principal non-negative root when an even root is intended.
- ☐ Expand or combine logarithms using their product, quotient and power rules; check positive arguments and valid bases.
- ☐ Change the logarithm base when useful, and solve exponential equations with common bases or logarithms.

1.8 A finite total from infinitely many terms

- ☐ Decide whether a geometric series converges by checking that the absolute value of its ratio is less than 1.
- ☐ Use the sum to infinity to find a total or an unknown parameter, and reject values that fail the convergence condition.

1 Number and algebra / continued

1.9 Expanding a positive integer power

- ☐ Use binomial coefficients to expand a power of a two-term expression or extract a required term or coefficient.
- ☐ Calculate combination coefficients from factorials, Pascal's triangle or technology, keeping powers and signs consistent.

1.10 Counting choices and extending binomial expansions (HL)

- ☐ Choose between multiplication, permutations and combinations by deciding whether order matters and whether choices are restricted.
- ☐ Generate terms of a binomial expansion with a negative or fractional power, first factoring out a suitable constant.
- ☐ State the interval where the infinite expansion is valid and use a suitable truncation for an approximation.
- ☐ Keep counting practice within the assessed scope: arrangements of identical objects and circular arrangements are outside this section.

1.11 Splitting an algebraic fraction (HL)

- ☐ Rewrite a proper rational expression as partial fractions and find the constants by substitution or matching coefficients.
- ☐ Handle denominators containing up to two distinct linear factors, and verify the result by recombining the fractions.

1.12 Locating complex numbers (HL)

- ☐ Work with $i^2 = -1$ and identify the real part, imaginary part and conjugate of a + bi.
- ☐ Plot a complex number on an Argand diagram and find its modulus and argument with the correct quadrant.

1.13 Choosing a useful complex-number form (HL)

- ☐ Convert between Cartesian form, r(cos θ + i sin θ), r cis θ and r e^(iθ).

1.13 Choosing a useful complex-number form (HL) - continued

- ☐ Choose an efficient form for addition, multiplication or division, then interpret the calculation geometrically.
- ☐ Connect multiplication and division with changes in modulus and argument, including rotations and stretches.

1.14 Complex powers, roots and polynomial equations (HL)

- ☐ Use conjugate pairs to find missing non-real roots of a polynomial with real coefficients.
- ☐ Apply de Moivre's theorem to complex powers and prove its positive-integer case by induction.
- ☐ Find all distinct nth roots of a complex number and represent their equal angular spacing on an Argand diagram.

1.15 Choosing a proof strategy (HL)

- ☐ Write a complete induction proof with a base case, a clear assumption and an inductive step that uses that assumption.
- ☐ Build a proof by contradiction, identifying exactly which assumption leads to the contradiction.
- ☐ Disprove a universal claim with a counterexample and explain why that example violates the claim.

1.16 Solving and classifying linear systems (HL)

- ☐ Solve up to three simultaneous linear equations in three unknowns using algebra and technology.
- ☐ Distinguish a unique solution, an inconsistent system and a system with infinitely many solutions.
- ☐ Express an infinite family of solutions using a free parameter and interpret parameter-dependent cases.

2 Functions

Move between equations, graphs and descriptions to understand how inputs determine outputs.

2.1 Describing straight lines

- ☐ Find a line's gradient and intercepts from points, an equation or a diagram.
- ☐ Build and rearrange line equations in gradient-intercept, point-gradient and general form.
- ☐ Use gradient relationships to construct parallel or perpendicular lines and interpret a gradient in its context.

2.2 Inputs, outputs and reversing a function

- ☐ Interpret function notation and determine sensible domains and ranges from a formula, graph or model.
- ☐ Explain how an inverse reverses a one-to-one function; interchange its domain and range.
- ☐ Relate a function and its inverse through reflection in $y = x$, without confusing an inverse with a reciprocal.

2.3 Turning function information into a graph

- ☐ Produce a labelled sketch from a formula, description or calculator display, showing the important shape and features.
- ☐ Use graphing technology with a suitable viewing window, including for sums or differences of functions.
- ☐ Distinguish a carefully scaled drawing from a sketch that communicates the essential behaviour.

2.4 Reading a graph with technology

- ☐ Locate roots, intercepts, turning points, extreme values and intersections accurately with graphing tools.
- ☐ Recognise symmetry and describe horizontal or vertical asymptotes from the graph.
- ☐ Check that the viewing window includes the features relevant to the question.

2.5 Combining and undoing functions

- ☐ Evaluate and form composite functions in the stated order, checking that intermediate outputs are allowed inputs.
- ☐ Find an inverse algebraically when the function is one-to-one and verify it by composition.
- ☐ Recognise the identity function and use the domain-range relationship when describing inverse functions.

2.6 Three useful views of a quadratic

- ☐ Move between expanded, factorised and completed-square forms of a quadratic.
- ☐ Read the intercepts, vertex and axis of symmetry from a suitable form and sketch the parabola.
- ☐ Explain how the leading coefficient affects the direction and vertical stretch of the graph.

2.7 Quadratic roots and solution intervals

- ☐ Solve a quadratic using factorisation, completing the square or the quadratic formula, choosing an efficient method.
- ☐ Solve quadratic inequalities and show the correct intervals, including or excluding endpoints as required.
- ☐ Use the discriminant to decide how many real roots occur and to find parameter conditions.

2.8 Reciprocals and linear-over-linear functions

- ☐ Explain the shape and self-inverse property of $y = 1/x$.
- ☐ Sketch a rational function with a linear numerator and denominator, identifying intercepts and excluded input values.
- ☐ Find and label its vertical and horizontal asymptotes, checking any factor cancellation before claiming an asymptote.

2 Functions / continued

2.9 Exponential and logarithmic graphs

- ☐ Sketch exponential and logarithmic functions, including base e, with their domains, ranges, intercepts and asymptotes.
- ☐ Explain the inverse relationship between these families and compare growth or decay for different valid bases.
- ☐ Rewrite a^x as e^(x ln a) and use inverse identities to simplify expressions.

2.10 Finding solutions in mathematical models

- ☐ Choose an algebraic, graphical or numerical method to solve an equation, including equations requiring technology.
- ☐ Use a substitution where it turns an exponential equation into a familiar algebraic form.
- ☐ Interpret solutions in a real situation, respecting the model's domain and the required accuracy.

2.11 Moving and stretching graphs

- ☐ Describe translations, reflections in either axis, and vertical or horizontal stretches using function notation.
- ☐ Apply a sequence of transformations in the correct order and track key coordinates, domains, ranges and asymptotes.
- ☐ Recognise that f(qx) gives a horizontal stretch with factor 1/q for q > 0; the general combined input ax + b belongs to the HL extension.

2.12 Reading and factoring polynomials (HL)

- ☐ Connect a polynomial's factors and roots with its graph, including repeated roots.
- ☐ Use the factor and remainder theorems to find unknown coefficients, test factors or solve polynomial equations.
- ☐ Use the sum and product of all roots, including complex roots, to relate solutions to the coefficients.

2.13 Rational graphs with quadratic terms (HL)

- ☐ Analyse rational functions with a linear numerator over a quadratic denominator, or a quadratic numerator over a linear denominator.
- ☐ Find intercepts, domain restrictions and horizontal, vertical or oblique asymptotes, using algebra to support the sketch.
- ☐ Use technology to investigate behaviour and check that a sketch shows each branch correctly.

2.14 Symmetry, repetition and invertibility (HL)

- ☐ Test whether a function is even or odd algebraically and relate the result to graph symmetry.
- ☐ Identify periodic behaviour and explain what a period means.
- ☐ Restrict a domain when necessary to obtain an inverse; find that inverse and recognise self-inverse functions.

2.15 Comparing two functions (HL)

- ☐ Find where one function lies above or below another by studying their difference or their graphs.
- ☐ Solve polynomial inequalities up to degree 3 analytically and express the answer with correct interval endpoints.
- ☐ Use technology for more complicated comparisons and respect discontinuities and domain restrictions.

2.16 Building new graphs from an existing graph (HL)

- ☐ Predict the graphs of |f(x)|, f(|x|), 1/f(x) and [f(x)]² from the behaviour of f.
- ☐ Handle combined input transformations f(ax + b), tracking fixed points, intercepts and asymptotes where relevant.
- ☐ Solve modulus equations and inequalities through appropriate cases, graphs or technology, checking the original conditions.

3 Geometry and trigonometry

Use shapes, angles and vectors to connect algebra with position, distance and motion.

3.1 Measuring three-dimensional objects

- ☐ Calculate the distance and midpoint between points given by three coordinates.
- ☐ Find volumes and surface areas of solids, including right pyramids, right cones, spheres, hemispheres and combined shapes.
- ☐ Identify useful right-angled triangles to find lengths and angles between intersecting lines or between a line and a plane.

3.2 Choosing a triangle method

- ☐ Choose sine, cosine or tangent for a right-angled triangle and use an inverse ratio when finding an angle.
- ☐ Choose the sine rule or cosine rule for an appropriate non-right-angled triangle.
- ☐ Calculate a triangle's area from two sides and their included angle; label a supporting diagram.

3.3 Turning a situation into a triangle

- ☐ Translate a written description into a clear diagram with the relevant lengths and angles marked.
- ☐ Solve problems involving bearings, elevation, depression and distance using Pythagoras or trigonometry.
- ☐ Interpret the result with suitable units and check that it matches the geometry.

3.4 Angles measured by arc length

- ☐ Convert between degrees and radians, using exact multiples of π when appropriate.
- ☐ Use radian measure to calculate arc lengths, sector areas and unknown circle dimensions.

3.5 Extending trigonometry around the circle

- ☐ Use the unit circle to determine signs, related angles and exact values based on 0, π/6, π/4, π/3 and π/2.
- ☐ Connect tangent with sine divided by cosine and with the gradient of a line through the origin.
- ☐ Check for two possible triangles when applying the sine rule in an ambiguous case.

3.6 Using identities instead of finding an angle

- ☐ Use $\sin^2 \theta + \cos^2 \theta = 1$ to connect ratios and select signs from the angle's quadrant.
- ☐ Apply the sine and cosine double-angle identities to simplify expressions or determine exact values.
- ☐ Find a related trigonometric ratio without first calculating the angle.

My next practice task

My next practice task

3 Geometry and trigonometry / continued

3.7 Modelling a repeating pattern

- ☐ Sketch sine, cosine and tangent graphs and identify their periods, zeros and relevant asymptotes.
- ☐ Interpret amplitude, horizontal shift, vertical shift and period in a transformed trigonometric model.
- ☐ Build or use a sinusoidal model for a repeating situation, matching units and interpreting parameters in context.

3.8 Finding every trigonometric solution in an interval

- ☐ Solve trigonometric equations by algebra or graphing and list all solutions in the specified finite interval.
- ☐ Use identities or substitution to reduce an equation to a quadratic in a trigonometric ratio.
- ☐ Check endpoints, angle units and rejected values; an unrestricted general solution is outside this section's requirements.

3.9 Reciprocal and inverse trigonometric functions (HL)

- ☐ Use sec, cosec and cot as reciprocal ratios, and work with their associated Pythagorean identities.
- ☐ Distinguish inverse trigonometric functions from reciprocals and use their principal ranges correctly.
- ☐ Sketch arcsin, arccos and arctan with accurate domains, ranges and key features.

3.10 Combining angles (HL)

- ☐ Use compound-angle identities to simplify expressions, establish results and find exact values.
- ☐ Derive double-angle identities from the compound-angle formulae and include the double-angle identity for tangent.

3.11 Explaining trigonometric symmetry (HL)

- ☐ Relate values at transformed angles using unit-circle reasoning, identities and graph symmetry.
- ☐ Use these relationships to simplify expressions and justify equivalent trigonometric forms.

3.12 Representing position and displacement (HL)

- ☐ Write vectors using components or i, j and k; distinguish a point's position vector from a displacement between points.
- ☐ Add, subtract and scale vectors algebraically and geometrically, including zero, opposite and parallel vectors.
- ☐ Find magnitudes and unit vectors and use vector relationships to prove geometrical results.

My next practice task

My next practice task

3 Geometry and trigonometry / continued

3.13 Angles from a scalar product (HL)

- ☐ Calculate a dot product using components or magnitudes and an included angle.
- ☐ Use scalar products to find angles and test perpendicularity or parallelism for non-zero vectors.
- ☐ Apply the algebraic properties of the dot product to simplify vector expressions and justify results.

3.14 Describing a line in space (HL)

- ☐ Write and convert a line's vector, parametric and Cartesian equations, identifying a point and a direction vector.
- ☐ Use direction vectors to calculate the angle between lines.
- ☐ Interpret a time parameter as motion at constant velocity and connect the velocity vector's magnitude with speed.

3.15 Deciding how two lines meet (HL)

- ☐ Classify a pair of lines as coincident, parallel, intersecting or skew.
- ☐ Find an intersection using independent parameters and verify the point in both line equations.
- ☐ Explain why non-parallel lines in three dimensions need not intersect.

3.16 Perpendicular directions and area (HL)

- ☐ Calculate a cross product, determine its direction and use its magnitude to connect with the included angle.
- ☐ Apply cross-product properties, including the sign change when reversing the order.
- ☐ Use the cross product to find a normal vector, test parallelism and calculate triangle or parallelogram area.

3.17 Describing a plane (HL)

- ☐ Construct a plane from a point and two independent directions, or from a point and a normal vector.
- ☐ Move between parametric vector, scalar-product and Cartesian forms of the plane equation.
- ☐ Check whether a point or a direction is compatible with the plane.

3.18 Intersections and angles involving planes (HL)

- ☐ Find and interpret intersections of a line with a plane, two planes, or three planes by solving suitable equations.
- ☐ Recognise when an intersection is empty, a point, a line or an entire plane.
- ☐ Use normal and direction vectors to find plane-plane and line-plane angles, taking the correct complementary angle when needed.

My next practice task

My next practice task

4 Statistics and probability

Critique data, quantify uncertainty and choose models that answer a clearly stated question.

4.1 Asking whether data can be trusted

- ☐ Distinguish populations from samples and discrete from continuous data; assess the credibility of the collection process.
- ☐ Compare simple random, systematic, stratified, quota and convenience sampling, identifying likely sources of bias.
- ☐ Investigate missing values and recording mistakes; identify outliers using the 1.5 IQR fences without automatically discarding valid observations.

4.2 Displaying and comparing distributions

- ☐ Organise discrete or continuous data in frequency tables and equal-width frequency histograms.
- ☐ Construct and read cumulative-frequency graphs to estimate medians, quartiles, percentiles and spread.
- ☐ Draw box plots with marked outliers and compare centre, spread and symmetry; consider whether a roughly normal shape is plausible.
- ☐ Use equal class widths for histogram work here; frequency-density histograms are outside the assessed section.

4.3 Choosing numerical summaries

- ☐ Calculate and interpret mean, median and mode; estimate a grouped mean using class midpoints and identify the modal class.
- ☐ Use technology for quartiles, variance and standard deviation, and interpret these alongside the interquartile range.
- ☐ Predict how adding a constant or multiplying every observation changes measures of centre and spread.
- ☐ Recognise that different quartile conventions can produce slightly different answers for discrete data.

4.4 Predicting y from x

- ☐ Describe a scatter plot's linear association and use technology to obtain and interpret Pearson's correlation coefficient.
- ☐ Draw a sensible best-fit line through the mean point and obtain the y-on-x regression equation with technology.
- ☐ Interpret the slope and intercept and make appropriate predictions of y, treating extrapolation and reverse prediction cautiously.
- ☐ Explain why strong correlation does not establish causation and why Pearson's r can mislead for a non-linear pattern.

4.5 Building a probability model

- ☐ Describe the sample space and events for a trial, checking whether the outcomes really are equally likely.
- ☐ Calculate theoretical probabilities and complements, and compare them with experimental relative frequencies.
- ☐ Use probability to predict an expected count over repeated trials, without interpreting that count as a guarantee.

4.6 Combining and conditioning events

- ☐ Choose a tree, Venn diagram, outcome table or sample-space diagram to organise a multi-stage probability problem.
- ☐ Calculate probabilities involving inclusive 'or', 'and', complements and conditional information.
- ☐ Distinguish independence from mutual exclusivity and update probabilities correctly when sampling without replacement.

4.7 A random quantity and its long-run average

- ☐ Build or interpret a discrete probability distribution from a table or rule and ensure its probabilities total 1.

4 Statistics and probability / continued

4.7 A random quantity and its long-run average - continued

- ☐ Calculate an expected value by weighting each possible value with its probability.
- ☐ Interpret expected net gain, including entry costs, to judge whether a game is fair.

4.8 Counting successes with a binomial model

- ☐ Check that a fixed number of independent trials each has two outcomes and the same success probability.
- ☐ Use binomial technology for individual, cumulative and interval probabilities, handling 'at least' and 'more than' carefully.
- ☐ Find and interpret the model's mean and variance, np and np(1 − p).

4.9 Areas under a normal curve

- ☐ Sketch a normal model with its mean and standard deviation and use symmetry and the approximate 68%, 95%, 99.7% pattern.
- ☐ Translate a probability statement into a shaded region and calculate its area with technology.
- ☐ Use inverse-normal technology to find a boundary or percentile when the mean and standard deviation are known.

4.10 Predicting x from y

- ☐ Find the x-on-y regression equation with technology and use it to predict x from a specified y.
- ☐ Choose the regression direction to match the variable being predicted; do not treat the two regression lines as interchangeable.

4.11 Checking independence with probability formulae

- ☐ Apply the conditional-probability formula to connect a restricted sample space with an intersection probability.

4.11 Checking independence with probability formulae - continued

- ☐ Test independence using $P(A \cap B) = P(A)P(B)$ or an equivalent conditional-probability comparison, where defined.
- ☐ Explain the conclusion in the context of the two events.

4.12 Standard scores and unknown normal parameters

- ☐ Convert between a value and its z-score and interpret the sign and size of that score.
- ☐ Use inverse-normal results to form equations for an unknown mean, standard deviation or both.
- ☐ Solve the parameter equations and check that the standard deviation is positive and the probabilities are consistent.

4.13 Reversing a conditional probability (HL)

- ☐ Use Bayes' theorem to revise the probability of a source or cause after observing an outcome.
- ☐ Build the denominator from all relevant cases in a partition of up to three events, using a tree when helpful.

4.14 Comparing discrete and continuous random variables (HL)

- ☐ Calculate $E(X)$, $E(X^2)$, variance and standard deviation from a discrete distribution or a continuous density, including piecewise densities.
- ☐ Check that a density is non-negative with total area 1, then use integrals for interval probabilities; a density value itself may exceed 1.
- ☐ Find a continuous distribution's median from cumulative area and its mode from the maximum density.
- ☐ Interpret fair net gains and use $E(aX + b) = aE(X) + b$ and $Var(aX + b) = a^2Var(X)$ for transformed variables.

5 Calculus

Connect local rates of change with accumulated change, then use both to analyse functions and models.

5.1 From a changing secant to an instantaneous rate

- ☐ Estimate a limit from a table or graph and describe what value the function approaches.
- ☐ Explain a derivative as a tangent gradient or an instantaneous rate, using notation suited to the variables.
- ☐ Connect the tangent gradient with a limiting sequence of secant gradients; formal limit techniques are reserved for HL.

5.2 Where a function rises or falls

- ☐ Use the sign of the first derivative to identify intervals of increasing or decreasing behaviour.
- ☐ Relate positive, negative and zero gradients to the graphs of a function and its derivative.

5.3 Differentiating integer powers

- ☐ Apply the power rule to constant multiples of integer powers, including negative powers after suitable rewriting.
- ☐ Differentiate sums of these terms and simplify the resulting derivative.

5.4 Equations touching and crossing a curve

- ☐ Find the gradient at a specified point using a derivative or technology, then write the tangent equation.
- ☐ Find the normal equation using perpendicularity, recognising horizontal and vertical special cases.

5.5 Recovering a function and measuring area

- ☐ Reverse the power rule for sums of integer powers other than x^−1, including the arbitrary constant.
- ☐ Use a given function value to determine that constant.
- ☐ Write a definite integral for the area beneath a positive curve and evaluate it with technology, connecting it with antiderivatives.

5.6 Choosing a differentiation rule

- ☐ Differentiate rational powers, sine, cosine, e^x and ln x, including their sums and constant multiples.
- ☐ Identify inner and outer functions and apply the chain rule to a composite expression.
- ☐ Use product and quotient rules when required, combining rules and simplifying carefully.

5.7 Reading a function through its derivatives

- ☐ Calculate second derivatives and use either $f''(x)$ or d^2y/dx^2 notation correctly.
- ☐ Connect the shapes of f, f′ and f″, including how changes in gradient show on the derivative graphs.

5.8 Classifying key points and finding an optimum

- ☐ Find stationary points and classify local maxima or minima using a sign change in f′ or a valid second-derivative test.

My next practice task

My next practice task

5 Calculus / continued

5.8 Classifying key points and finding an optimum - continued

- ☐ Distinguish concave-up and concave-down intervals and confirm a change in concavity at an inflection point.
- ☐ Set up and solve an optimisation problem within a meaningful domain, comparing relevant boundary values.
- ☐ Recognise that $f'' = 0$ alone does not prove an inflection point, and that an inflection point need not be stationary.

5.9 Connecting position, velocity and acceleration

- ☐ Move between displacement, velocity and acceleration using differentiation or integration and any initial conditions.
- ☐ Distinguish speed from velocity and identify changes of direction.
- ☐ Calculate displacement from the integral of velocity and total distance from the integral of speed, splitting at sign changes when useful.

5.10 Recognising antiderivative patterns

- ☐ Integrate rational powers, sine, cosine, exponentials and $1/x$, using $\ln|x|$ for the last case.
- ☐ Integrate these functions with a linear inner expression, including the required constant factor.
- ☐ Recognise a multiple of an inner derivative and use the reverse chain rule or an equivalent substitution.

5.11 Definite integrals and enclosed regions

- ☐ Evaluate a definite integral from an antiderivative, or use technology when an analytic antiderivative is unsuitable.
- ☐ Separate signed accumulation from geometrical area and split an interval where the curve changes sign.
- ☐ Find areas between curves by locating intersections and integrating the appropriate difference, writing the setup before calculating.

5.12 Limits, first principles and higher derivatives (HL)

- ☐ Describe convergence, divergence, continuity and differentiability informally and explain how these ideas connect.
- ☐ Use the limiting difference quotient to derive the derivative of a polynomial.
- ☐ Calculate and interpret higher derivatives using nth-derivative notation; formal continuity or differentiability tests are outside the stated examination requirements.

5.13 Resolving indeterminate limits (HL)

- ☐ Recognise a $0/0$ or ∞/∞ quotient before applying l'Hôpital's rule, repeating it where valid and necessary.
- ☐ Use suitable Maclaurin expansions as an alternative way to evaluate limits at finite points or infinity.
- ☐ Check the transformed expression and the existence of the resulting limit instead of treating the rule as automatic.

My next practice task

My next practice task

5 Calculus / continued

5.14 Implicit relationships and linked rates (HL)

- ☐ Differentiate a relation that defines y implicitly, applying the chain rule to every term involving y.
- ☐ Connect rates of change through an equation between changing quantities and interpret signs and units.
- ☐ Solve optimisation problems requiring implicit differentiation or the chain rule and check possible endpoint optima.

5.15 Extending the derivative and integral toolkit (HL)

- ☐ Differentiate tan, sec, cosec, cot, a^x, $\log_a x$, arcsin, arccos and arctan.
- ☐ Recognise and integrate their derivative patterns, including compositions with a linear expression.
- ☐ Rearrange suitable integrands using partial fractions and interpret an indefinite integral as a family of curves.

5.16 Substitution and integration by parts (HL)

- ☐ Carry out a substitution consistently, changing the differential and either the limits or the final variable.
- ☐ Choose and apply integration by parts, including repeated applications and cases where the original integral reappears.
- ☐ Recognise the examination scope: a substitution is supplied when the integrand is not a direct reverse-chain-rule pattern.

5.17 Integrating across and rotating a region (HL)

- ☐ Set up an integral with respect to y for a region bounded by a curve and the y-axis.
- ☐ Find volumes formed by rotating a region about the x-axis or y-axis, using appropriate outer and inner radii.
- ☐ Choose bounds and the variable of integration to match the geometry, then interpret the volume in cubic units.

5.18 Solving first-order differential equations (HL)

- ☐ Generate an approximate solution to dy/dx = f(x, y) with Euler's method and a stated constant step size.
- ☐ Separate variables and integrate, using an initial condition when supplied and retaining relevant constant solutions.
- ☐ Solve a homogeneous equation dy/dx = f(y/x) using y = vx and the product rule.
- ☐ Use an integrating factor for a linear equation y′ + P(x)y = Q(x), and check the resulting solution.

5.19 Approximating functions with power series (HL)

- ☐ Obtain Maclaurin expansions for e^x, sin x, cos x, arctan x, ln(1 + x) and (1 + x)^p with rational p.
- ☐ Generate related expansions through substitution, multiplication, differentiation or integration, keeping the required order of terms.
- ☐ Use a differential equation to determine series coefficients and apply an expansion only within an appropriate validity range.

My next practice task

My next practice task